114. Determine whether the following arcs are rectifiable, and find the arc lengths:

(a) \( x(t) = \begin{pmatrix} r \cos t \\ r \sin t \\ ht^2 \end{pmatrix}, \ t \in [0, 2\pi] \ (r, h > 0 \text{ are fixed}); \)

(b) \( x(t) = \begin{pmatrix} t \\ t \sin \left(\frac{1}{t}\right) \end{pmatrix}, \ t \in (0, 1], \ x(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \)

115. Consider \( x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ x_n = \begin{pmatrix} 1/2^n \\ a \end{pmatrix} \) for \( n \in \mathbb{N} \), \( y = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \), and the set \( M \) defined as \( \bigcup_{n \in \mathbb{N}} x_0 x_n \cup \{y\} \), where \( ab \) denotes the line connecting the points \( a \) and \( b \). Graph \( M \). Is \( M \) open? Is \( M \) connected? Is \( M \) arcwise connected?

116. Consider two nonempty convex sets \( M_1, M_2 \subset \mathbb{R}^n \). Show:

(a) \( M_1 - M_2 := \{x - y : x \in M_1, y \in M_2\} \) is convex.

(b) If \( M_1 \cap M_2 = \emptyset \), then there exist \( p_0 \in \mathbb{R}^n \setminus \{0\} \) and \( \alpha \in \mathbb{R} \) with
\[ p_0^T x \geq \alpha \geq p_0^T y \text{ for all } x \in M_1 \text{ and all } y \in M_2. \]

117. Let \( A \) be a symmetric \( n \times n \)-matrix and define \( f : \mathbb{R}^n \to \mathbb{R} \) by \( f(x) = x^T A x \).

Show that \( f \) is convex iff the matrix \( A \) is positive semidefinite.

118. Determine whether the following functions \( f : \mathbb{R}^2 \to \mathbb{R} \) are convex:

(a) \( f(x, y) = x^2 + 3y^2 - 3xy; \)

(b) \( f(x, y) = 2x^2 - y^2 + 4xy. \)

119. Suppose \( M \subset \mathbb{R}^n \) is convex. Show that a function \( f : M \to \mathbb{R} \) is convex iff the so-called epigraph \( \{(x, y) \in M \times \mathbb{R} : f(x) \leq y\} \) is convex.

120. Suppose \( M \subset \mathbb{R}^n \) is convex and \( f : M \to \mathbb{R} \) is strictly convex. Assume that there exists \( x_0 \in M \) with \( f(x_0) = \max f(M) \). Prove that \( x_0 \in \partial M \).

121. Suppose \( M \subset \mathbb{R}^n \) is closed and convex, and let \( x_0 \in \mathbb{R}^n \). Prove that there exists exactly one \( a \in M \) with \( d(x_0, M) = \|x_0 - a\| \).