122. Work on Problems 1–3 of Chapter 11.1 in the textbook.

123. Find, where existent, all partial derivatives and the gradient of the following functions \( f \):

(a) \( f : \mathbb{R}^2 \to \mathbb{R}, f(x, y) = \begin{cases} \frac{x^3}{x^2 + y^2} & \text{if } x^2 + y^2 > 0 \\ 0 & \text{if } x = y = 0 \end{cases} \)

(b) \( f : \mathbb{R}^2 \to \mathbb{R}^3, f(x, y) = \begin{pmatrix} \ln(1 + x^2 + y^2) \\ \cos(x^2 y) \\ 1 - \sqrt{1 + x^2 + y^2} \end{pmatrix} \)

(c) \( f : \mathbb{R}^3 \to \mathbb{R}^3, f(r, \phi, z) = \begin{pmatrix} r \cos \phi \\ r \sin \phi \\ z \end{pmatrix} \).

124. Let \( f : \mathbb{R}^3 \to \mathbb{R} \) be defined by \( f(x, y, z) = x^2 + z e^y \). Calculate \( \frac{\partial f}{\partial v}(1, 0, 1) \) for \( v = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \). For which vectors \( v \in \mathbb{R}^3 \) with \( \|v\| = 1 \) is \( \left| \frac{\partial f}{\partial v}(1, 1, 1) \right| \) maximal? For which vectors \( w \in \mathbb{R}^3 \) with \( \|w\| = 1 \) is \( \left| \frac{\partial f}{\partial w}(1, 1, 1) \right| \) minimal? What is the relationship between \( v \) and \( w \)?

125. Find \( \int_{-\infty}^{\infty} \cos(xt)e^{-t^2/2} \, dt \) for \( x \in \mathbb{R} \).

126. Calculate \( \int_{0}^{1} \int_{1}^{2} e^{xy} \, x \, dx \, dy \).

127. A function \( f : \mathbb{R}^2 \to \mathbb{R} \) is called harmonic provided that it has second-order partial derivatives and \( \frac{\partial f}{\partial x^2}(x, y) + \frac{\partial f}{\partial y^2}(x, y) = 0 \) for all \( (x, y) \in \mathbb{R}^2 \). Which of the following functions are harmonic?

(a) \( f(x, y) = e^x \cos y \);
(b) \( g(x, y) = x^2 - y^2 \);
(c) \( h(x, y) = x^2 - y^3 \).