Exam #1, Math 315, Dr. M. Bohner, Feb 16, 2005. | Name: |

Let $P = \{x_0, x_1, \dots, x_n\}$ be a partition of $[a,b]$ and let $f, g : [a,b] \to \mathbb{R}$. The norm of P is defined as $\|P\| =$ ______. We also define $\bigvee(P,f) =$ ______. A Riemann–Stieltjes sum for f with respect to g is defined by $S(P,\xi,f,g) =$ ______. A function f is said to be of ______ provided its total variation on $[a,b]$, defined by $\bigvee_a^b f =$ ______, is less than infinity. For example $\bigvee_0^{3\pi} \sin =$ ______. We showed that for $f \in \mathrm{BV}[a,b]$, both v_f and ______ are non ______ functions. A function f is in $\mathrm{BV}[a,b]$ if and only if it can be written as ______. The function f is called Riemann–Stieltjes integrable with respect to g if there exists a number $J \in \mathbb{R}$ with the following property: ______. The Fundamental Inequality for Riemann–Stieltjes integrals says ______.

If f is constant, then $\int_a^b f dg =$ ______, and if g is constant, then $\int_a^b f dg =$ ______. If f is Riemann integrable on $[a,b]$ and if g' is continuous on $[a,b]$, then $\int_a^b f dg =$ ______. For example, $\int_0^1 x d(x^2) =$ ______. The "integration by parts formula" for Riemann–Stieltjes integrals reads ______. For example, $\int_0^4 x^2 d[x] =$ ______. The integration by parts formula can also be used to prove Euler's summation formula, which says ______. The main existence theorem for Riemann–Stieltjes integrals says that ______ imply $f \in \mathcal{R}(g)$. If α is a nondecreasing function, then we define the lower and upper sums L and U by

______, while

the lower and upper Riemann–Stieltjes integrals are defined by

______.

Next, a sequence of functions $\{f_n\}_{n\in\mathbb{N}}$ defined on E is called uniformly convergent on E if ____. The function sequence $\{f_n\}$, defined by $f_n(x) = x^n$, ____ uniformly convergent on $[0, 1]$ while it ____ uniformly convergent on $[0, 1/2]$. The Cauchy criterion for uniform convergence says ____. The ____ M-Test says that $\sum_{k=1}^{\infty} g_k$ converges uniformly provided ____. The three main theorems on continuity, integrability, and differentiability of the limit functions say

____,

____,

____.

The space $\mathrm{C}(X)$, together with the supnorm defined by $\|f\|_\infty =$ ____, is a Bana ____ space. Let $\mathcal{F}$ be a family of real-valued functions defined on E. The family $\mathcal{F}$ is called equicontinuous on E provided

____. A sufficient criterion for equicontinuity is that

____. The family $\mathcal{F}$ is called pointwise bounded on E if ____, and it is called uniformly bounded on E if ____. The Arz ____ theorem says

____. The Weierstraß approximation theorem says that ____. The polynomials can be chosen as the Bernstein polynomials, which are defined as

$B_n(f; x) =$ ____.