Exam #2, Math 315, Dr. M. Bohner, Mar 14, 2005. Name: ____________

A function f is said to be represented by a power series around a provided $f(x) =$ ____________.

If R is the radius of convergence of a power series, then it converges uniformly on ____________.

Within the radius of convergence, we have the formula $f^{(n)}(x) =$ ____________. By using these formulas, we can find $\sum_{k=1}^{\infty} \frac{1}{k2^k} =$ ____________ and $\sum_{k=1}^{\infty} \frac{1}{2^k} =$ ____________. Abel's theorem says that ____________.

Taylor's theorem says that ____________.

Taylor's theorem and Abel's theorem can be used to find $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} =$ ____________. We defined the exponential function E by $E(z) =$ ____________ for all $z \in \mathbb{C}$. We defined the number e as ____________. We defined the trigonometric functions S and C by $S(x) =$ ____________ and $C(x) =$ ____________. We defined the number π as ____________. A sequence $\{\phi_n\}_{n\in\mathbb{N}}$ is called an orthogonal system of functions on $[a, b]$ if ____________. It is called orthonormal on $[a, b]$ if in addition ____________. In this case the Fourier coefficients of a function f are defined by ____________. The Fourier series of an even function can be written as ____________ and the Fourier series of an odd function can be written as ____________. Bessel's inequality says ____________. The Dirichlet kernel is defined as

$D_N(x) =$ ______, and we showed that it can also be written as ______.

The localization theorem says that ______.

The Gamma function is defined by $\Gamma(x) =$ ______. The three properties that determine the Gamma function uniquely are ______, ______, and ______. We also showed that $\Gamma(1/2) =$ ______. The Beta function is defined by $B(x, y) =$ ______, and its relation with the Gamma function is ______. Stirling's formula says that ______.

The volume of an interval $I = [a_1, b_1] \times \cdots \times [a_N, b_N]$ is defined by $|I| =$ ______. The outer measure of any set $A \subset \mathbb{R}^N$ is defined by $\mu^*(A) =$ ______. For $A = \{a\}$ we have $\mu^*(A) =$ ______. If I is an interval, then we have $\mu^*(I) =$ ______. The outer measure is monotone, i.e., ______, and it is subadditive, i.e., ______. We also showed that in the following two cases the outer measure is even additive: ______ and ______. A set $A \subset \mathbb{R}^N$ is said to be Lebesgue measurable provided ______.