Exam #3, Math 315, Dr. M. Bohner, Apr 29, 2005.

Name:

The volume of an interval $I = [a_1, b_1] \times \cdots \times [a_N, b_N]$ is defined by $|I| =$ ______. The outer measure of any set $A \subset \mathbb{R}^N$ is defined by

$\mu^*(A) =$ ______.

For $A = \{a\}$ we have $\mu^*(A) =$ ______. If I is an interval, then we have $\mu^*(I) =$ ______.

The outer measure is monotone, i.e., ______, and it is subadditive, i.e., ______. A set $A \subset \mathbb{R}^N$ is said to be Lebesgue measurable provided ______.

An example of a not Lebesgue measurable set is ______.

A set $\mathcal{A} \subset \mathcal{P}(X)$ is called a σ-algebra in X if ______.

A triple $(X, \mathcal{A}, \mu)$ is called a measure space if ______.

A measure space is called complete provided ______. We showed the following two results about continuity of the measure: ______. A function $f : X \to \overline{\mathbb{R}}$ is called measurable if ______. A characteristic function K_E is measurable iff ______. If f is measurable, then so is $|f|$, $f^+ =$ ______, and $f^- =$ ______. Two functions are said to be equal almost everywhere if ______. A function is called summable if ______. For a summable function f we defined the following objects: $A_\infty(f) =$ ______, $A_{nk}(f) =$ ______, and $s_n(f) =$ ______. We then defined $\int_X f d\mu =$ ______.

Please list here the main properties of the s_n together with their consequences for the integral:

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If $x \geq 0$ and A is measurable, then [$\int_A c d\mu =$]. We proved the following characterization of functions that are summable and almost everywhere equal to zero: [$\Longleftrightarrow$].

Chebyschev's inequality says that [ ]. If f is the Dirichlet function, then [$\int_X f d\mu =$]. If $(X, \mathcal{A}, \mu)$ is a measure space, then we defined a new measure space $(x, \mathcal{A}, \nu)$ as follows: [ ]. Please state here the famous monotone convergence theorem by Beppo Levi:

[ ].

Fatou's lemma says that [ ].

A function $f : X \to \overline{\mathbb{R}}$ is called integrable if it is measurable and satisfies [ ], and in this case we define [$\displaystyle\int_X f d\mu =$]. Please state here the famous dominated convergence theorem by Lebesgue:

[ ].

For $p \geq 1$ we defined [$L^p(I) =$] and showed the following inequalities:

[(Hölder)]

[(Minkowski)].

The norm of an L^p-function f is defined by [$\|f\| =$]. With respect to this norm, the spaces L^p are [ ].