92. Show that for $\alpha > 90$. Suppose $(X, \mathcal{A}, \mu)$ is a measure space, $f : X \to \mathbb{R}$ is measurable, and $A \in \mathcal{A}$ with $\mu(A) < \infty$. Show that $f$ is integrable on $A$ if $\sum_{n=1}^{\infty} n\mu(A_n) < \infty$, where $A_n = \{x \in A : n - 1 \leq |f(x)| < n\}$.  
93. Suppose $(X, \mathcal{A}, \mu)$ is a measure space and $f : X \to \mathbb{R}$ is integrable on $X$. Show that for each $\varepsilon > 0$ there exists a simple function $e$ such that $\int_X |f - e|d\mu < \varepsilon$.  
94. A sequence $f$ is said to be convergent almost everywhere on $X$ if $\lim_{n \to \infty} f_n(x) = f(x)$ for all $x \in X$ almost everywhere.  
95. For $\nu : \mathcal{A} \to [0, \infty]$ by $\nu(A) = \int_A f d\mu$ for all $A \in \mathcal{A}$. Show that any summable function $h$ satisfies $\int_A h d\nu = \int_A h f d\mu$ for all $A \in \mathcal{A}$.  
96. Suppose $\nu$ is a signed measure on $(X, \mathcal{A})$ and $\nu^+(A) = \nu(\alpha A)$, $\nu^-(A) = \nu(\gamma A)$ (with $\alpha$ and $\gamma$ from the Hahn decomposition), and $|\nu(A)| = \nu^+(A) + \nu^-(A)$. Show the following:  
(a) $\nu^+$ and $\nu^-$ do not depend on $P$ and $N$;  
(b) At least one of the measures $\nu^+$ and $\nu^-$ is finite;  
(c) $\nu^+(A) = \sup\{\nu(B) : B \subseteq A, B \in \mathcal{A}\}$;  
(d) $-\nu^-(A) = \inf\{\nu(B) : B \subseteq A, B \in \mathcal{A}\}$;  
(e) $|\nu(A)| \leq |\nu|(A)$;  
(f) $\nu^+, \nu^-, \nu \ll |\nu|$:  
(g) $\nu^+, \nu^-, \nu \ll \mu$.  
98. Suppose $\nu$ is a signed measure on $(X, \mathcal{A})$ and $\nu^+(A) = \nu(\alpha A)$, $\nu^-(A) = \nu(\gamma A)$ (with $\alpha$ and $\gamma$ from the Hahn decomposition), and $|\nu(A)| = \nu^+(A) + \nu^-(A)$. Show the following:  
(a) $\nu^+$ and $\nu^-$ do not depend on $P$ and $N$;  
(b) At least one of the measures $\nu^+$ and $\nu^-$ is finite;  
(c) $\nu^+(A) = \sup\{\nu(B) : B \subseteq A, B \in \mathcal{A}\}$;  
(d) $-\nu^-(A) = \inf\{\nu(B) : B \subseteq A, B \in \mathcal{A}\}$;  
(e) $|\nu(A)| \leq |\nu|(A)$;  
(f) $\nu^+, \nu^-, \nu \ll |\nu|$:  
(g) $\nu^+, \nu^-, \nu \ll \mu$.