73. Show that the Cantor set is uncountable but has Lebesgue measure zero.

74. Define a relation $x \sim y$ if $x - y \in \mathbb{Q}$. Show that "$\sim$" is an equivalence relation. By the axiom of choice, there exists a set M that contains exactly one element (without loss of generality $M \subset [-1, 1]$) of each equivalence class. Show that M is not Lebesgue measurable. This is Vitali's example of a set which is not Lebesgue measurable.

75. Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $B \in \mathcal{A}$ be not empty. Define $\mathcal{A}^* = \{A \in \mathcal{A} : \ A \subset B\}$. Show that $(B, \mathcal{A}^*, \mu)$ is a measure space.

76. Let $X \neq \emptyset$ and $p : X \to [0, \infty]$. Define $\mu(A) = \sum_{x \in A} p(x)$, where a sum over uncountably many positive terms is ∞. Show that $(X, \mathcal{P}(X), \mu)$ is a measure space.

77. Assume $f_n : X \to \overline{\mathbb{R}}$ are all measurable, $n \in \mathbb{N}$. Show that $f = \inf_{n \in \mathbb{N}} f_n$ is also measurable.

78. Please prove the following rules about characteristic functions: $K_A \leq K_B$ if $A \subset B$, $A = B$ iff $K_A = K_B$, $K_\emptyset = 0$, $K_X = 1$, $K_{A^C} = 1 - K_A$, $\inf_{\omega \in \Omega} K_{A_\omega} = K_{\bigcap_{\omega \in \Omega} A_\omega}$ (how about the supremum?) $K_{\biguplus_{n \in \mathbb{N}} A_n} = \sum_{n \in \mathbb{N}} K_{A_n}$, $K_{A \cup B} = K_A + K_B - K_{AB}$, $K_{AB} = K_A K_B$.

79. Plot some of the sets A_{nk} for several values of n for the functions
    (a) $f(x) = x$;
    (b) $f(x) = \sqrt{x}$;
    (c) $f(x) = x^2$.