72. Determine the points where the Cauchy-Riemann equations are satisfied for

    (a) $f(z) = \mathrm{e}^z$;

    (b) $f(z) = \bar{z}$.

73. Determine v such that $f = u + \mathrm{i}v$ with $u(x + \mathrm{i}y) = 2x^3y - 2xy^3 + x^2 - y^2$ and $f(0) = \mathrm{i}$ satisfies the Cauchy-Riemann equations.

74. Show that the Laplacian operator is invariant under translations and rotations in the plane.

75. Find the transformed Laplacian operator in the plane when polar coordinates are introduced.

76. Find the transformed Laplacian operator in three dimensions when spherical coordinates are introduced.

77. Read Section 8.4 and work on all the exercises of this section.

78. Separate the variables to find solutions of the following partial difference equations:

    (a) $u(m+1, n) = u(m, n+1)$;

    (b) $u(m+1, n) = 4u(m, n+1)$;

    (c) $u(m+1, n) - 2u(m, n+1) - 3u(m, n) = 0$;

    (d) $u(m+2, n) = 4u(m, n+1)$.

79. Two players P and Q play a game where at each stage P wins a chip from Q with probability p and Q wins a chip from P with probability $q = 1 - p$. The game ends when one player is out of chips (and this player then looses). Let $u(m, n)$ denote the probability that P wins the game if P starts the game with m chips and Q starts with n chips.

    (a) Find $u(3, 0)$, $u(1, 1)$, $u(2, 1)$, and $u(2, 2)$.

    (b) Find a partial difference equation for u.

    (c) Solve the equation by substituting $u(m, n) = z(m)f(m + n)$.

    (d) Give the boundary conditions.

    (e) Find $u(m, n)$ for each $m, n \in \mathbb{N}_0$ (unless $m = n = 0$).

80. Work again on Problems 1–79 to be prepared for the final examination.