6. Consider the PDE $u_{x}+u_{t}=u$.
(a) Apply the geometric method to obtain an idea how the general solution looks like.
(b) Find the general solution.
(c) Find the solution $u$ with $u(0, t)=0$.
(d) Find the solution $u$ with $u(0, t)=e^{t}$.
(e) Find the solution $u$ with $u(0, t)=g(t)$, where $g$ is an arbitrary differentiable function.
(f) Find the solution $u$ with $u(x, 0)=g(x)$, where $g$ is an arbitrary differentiable function.
7. Consider the $\operatorname{PDE} u_{x}+u_{t}=u+e^{x-t}$.
(a) Apply the geometric method to obtain an idea how the general solution looks like.
(b) Find the general solution.
(c) Find the solution $u$ with $u(x, 0)=g(x)$, where $g$ is an arbitrary differentiable function.
(d) Find the solution $u$ with $u(x, 1)=g(x)$, where $g$ is an arbitrary differentiable function.
8. Find the solution of $u_{x}+u_{t}+u=e^{x+2 t}$ that satisfies $u(x, 0)=0$.
9. Find the solution of $2 u_{x}+3 u_{t}=4 u+x$ that satisfies $u(x, 0)=9 x^{2}$.
10. Find the general solutions of the following equations. Where are they defined?
(a) $x u_{x}+t u_{t}=0$;
(b) $x u_{x}+t u_{t}=t$;
(c) $x u_{x}+t u_{t}=t^{2}+x^{3}$;
(d) $\left(1+x^{2}\right) u_{x}+u_{t}=0$.
11. Find the solution of $\sqrt{1-x^{2}} u_{x}+u_{t}=0$ that satisfies $u(0, t)=t$.
12. Find the solution of $t u_{x}+x u_{t}=0$ that satisfies $u(0, t)=e^{-t^{2}}$.
