13. Transform the following PDEs into standard form. For each step, write down exactly the transformation that is needed. Also, determine whether the equation is hyperbolic, elliptic, or parabolic:
(a) $3 u_{x x}+4 u_{t t}-u=0$;
(b) $4 u_{x x}+u_{x t}+4 u_{t t}+u=0$;
(c) $u_{x x}+u_{t t}+3 u_{x}-4 u_{t}+25 u=0$;
(d) $u_{x x}+2 u_{x t}+u_{t t}+u_{x}-2 u_{t}+u=0$.
14. Find all second-order linear homogeneous PDEs in $x$ and $t$ that are left unchanged by any rotation of axes.
15. Determine $\alpha$ so that a rotation with $\alpha^{\mathrm{o}}$ transforms $u_{x t}=0$ into $u_{x x}-u_{t t}=0$. Find the general solutions of both PDEs.
16. Determine a transformation that transforms $u_{x x}-u_{t t}=0$ into $u_{t t}=d u_{x x}$, where $d>0$. Then find the general solution of $u_{t t}=d u_{x x}$.
17. Obtain the general solution of $3 u_{x x}+10 u_{x t}+3 u_{t t}=0$.
