

1. For  $u(x, t) = x^2 e^t - \sqrt{t} e^{-x}$ , find  $u_{xt}(\ln 2, 9)$ .
2. Use the geometric method to find the general solution of  $xu_x + t^2 u_t = x$ .
3. Solve  $u_{xx} - 2u_{xt} + u_{tt} = x + t$ ,  $u(x, 0) = \phi(x)$ ,  $u_t(x, 0) = \psi(x)$  by “factoring the operator”.
4. Prove: If  $\phi$  is a periodic function with period  $p$ , then the solution of the diffusion equation  $u_t = k u_{xx}$  (on the whole real line) together with  $u(x, 0) = \phi(x)$  is also periodic in  $x$  with period  $p$ .
5. Separate the variables for  $u_{xx} + 2u_x + u_t = 0$ ,  $u(0, t) = u(1, t) = 0$  and find all eigenvalues and eigenfunctions of the resulting eigenvalue problem.
6. Let  $f(x) = a$  for  $-\pi \leq x < 0$ ,  $f(0) = 0$ , and  $f(x) = b$  for  $0 < x \leq \pi$ . Find the Fourier series of  $f$  in  $[-\pi, \pi]$ . Does it converge pointwise to  $f$ ?
7. Perform separation of variables to solve  $u(m+1, n) - u(m, n+1) + u(m, n) = 0$ ,  $u(m, 0) = 2^m$ .
8. Solve  $u_t = u_{xx}$  in  $[0, 4]$  with  $u = 0$  at both ends and  $u(x, 0) = x(5-x)$ , using the forward difference scheme with  $\Delta x = 1$  and  $\Delta t = 0.25$  to find the approximate value of  $u(2, 1)$ .