

74. Prove the Cox–Ross–Rubinstein formula for European put options.
75. Use the previous problem to show that put-call parity holds.
76. Let  $S_0 = 50$ ,  $r = 0.005$ ,  $T = 1$ ,  $u = 1.01$ , and  $d = 0.99$ . Give the price of a European call with strike price 60 to be exercised after 50 time steps.
77. Consider an option on a non-dividend-paying stock where the stock price is \$40, the strike price is \$40, the risk-free rate is 4% per annum, the volatility is 30% per annum, and the time to maturity is 6 months. Use a two-step tree to value the option if it is a (a) European call; (b) European put; (c) American call; (d) American put.
78. Using a two-step tree, value an American put option on a futures contract, where the strike price and the futures price are \$50, the risk-free rate is 10%, the time to maturity is 6 months, and the volatility is 40% per annum.
79. The AUD is currently worth 0.61 USD, and this exchange rate has a volatility of 10% per annum. The domestic interest rate is 5% per annum and the foreign interest rate is 8% per annum. Value a 3-month American call with strike price of 0.6 using a three-step tree.
80. A company's cash position, measured in millions of dollars, follows a generalized Wiener process with a drift rate of 0.5 per quarter and a variance rate of 4.0 per quarter. How high does the company's initial cash position have to be for the company to have a less than 5% chance of a negative cash position by the end of one year?
81. A company's cash position, measured in millions of dollars, follows a generalized Wiener process with a drift rate of 0.1 per month and a variance rate of 0.16 per month. The initial cash position is 2.0.
  - (a) What are the probability distributions of the cash position after 1 month, 6 months, and 1 year?
  - (b) What are the probabilities of a negative cash position at the end of 6 months and 1 year?
  - (c) At what time in the future is the probability of a negative cash position greatest?
82. Consider a variable  $S$  that follows the process  $dS = \mu dt + \sigma dW$ . For the first three years,  $\mu = 2$  and  $\sigma = 3$ , for the next three years,  $\mu = 3$  and  $\sigma = 4$ . If the initial value of the variable is 5, what is the probability distribution of the value of the variable at the end of year 6?
83. Consider a non-dividend-paying stock with volatility 20% providing expected return of 10%. Use Monte Carlo simulation to estimate the stock price after 10 weeks, using the following random sample for  $\varepsilon$ : 0.52, 1.44, -0.86, 1.46, -0.69, -0.74, 0.21, -1.1, 0.73, 1.16, 2.56. The current stock price is \$100.
84. Suppose that a stock price has an expected return of 16% per annum and a volatility of 30% per annum. When the stock price at the end of a certain day is \$50, calculate the following:
  - (a) The expected stock price at the end of the next day.
  - (b) The standard deviation of the stock price at the end of the next day.