10. Let $1 \le p \le \infty$. Show that $H^p(\partial\Delta)$ are Banach spaces.

11. For $z \in \partial\Delta$, define $f(z) = \frac{1}{\sqrt{1-z}}$.
    (a) Is $f \in H^1(\partial\Delta)$?
    (b) Is $f \in H^2(\partial\Delta)$?

12. Let X, Y be vector spaces and $L : X \to Y$ be linear. Put $\operatorname{Ker}(L) = \{x \in X : L(x) = 0\}$. Show:
    (a) L is one-to-one if and only if $\operatorname{Ker}(L) = \{0\}$.
    (b) All solutions of $L(x) = y$ are given by $x_0 + \operatorname{Ker}(L)$, where $L(x_0) = y$.

13. (a) Let X be a vector space and $A, B : X \to X$ be linear such that $AB = BA$. Prove that $(A+B)^n = \sum_{k=0}^n \binom{n}{k} A^{n-k}B^k$ holds for all $n \in \mathbb{N}$.
    (b) Define D on the set of all sequences by $D(\xi_1, \xi_2, \xi_3, \dots) = (\xi_2 - \xi_1, \xi_3 - \xi_2, \xi_4 - \xi_3, \dots)$. Also define recursively $\Delta^0\xi_\nu = \xi_\nu$ and $\Delta^k\xi_\nu = \Delta^{k-1}\xi_{\nu+1} - \Delta^{k-1}\xi_\nu$ for all $k \in \mathbb{N}$. Use part (a) to show $\Delta^k\xi_0 = \sum_{\nu=0}^k (-1)^\nu \binom{k}{\nu}\xi_{k-\nu}$. Use this last formula to calculate $\sum_{i=0}^n (-1)^i \binom{n}{i}\binom{i+a}{m}$.

14. Find the norms of the following operators:
    (a) The forward and backward shift operators on l^2;
    (b) $D : l^\infty \to l^\infty$ (D defined as in the previous problem).

15. Find the norms of the following operators:
    (a) The integration operator on $C[a, b]$;
    (b) Kepler's operator $Q : C[a, b] \to \mathbb{R}$ defined by $Qx = \frac{b-a}{6}\{x(a) + 4x(\frac{a+b}{2}) + x(b)\}$;
    (c) Bernstein's operator $B_n : C[0, 1] \to C[0, 1]$ defined by $(B_n x)(t) = \sum_{k=0}^n x(\frac{k}{n})\binom{n}{k} t^k(1-t)^{n-k}$.

16. For a continuous function $k : [c, d] \times [a, b] \to \mathbb{C}$ define K by $(Kx)(t) = \int_a^b k(t, s)x(s)\mathrm{d}s$.
    (a) Is K linear?
    (b) Is $K : L^2(a, b) \to L^2(c, d)$ bounded?
    (c) Is $K : C[a, b] \to C[c, d]$ bounded? If so, find $\|K\|$.

17. Let A be an $n \times m$-matrix and define L by $L(x) = Ax$ for all $x \in \mathbb{R}^m$. Find $\|L\|$:
    (a) $L : (\mathbb{R}^m, \|\cdot\|_\infty) \to (\mathbb{R}^n, \|\cdot\|_\infty)$;
    (b) $L : (\mathbb{R}^m, \|\cdot\|_1) \to (\mathbb{R}^n, \|\cdot\|_1)$;
    (c) $L : (\mathbb{R}^m, \|\cdot\|_1) \to (\mathbb{R}^n, \|\cdot\|_\infty)$;
    (d) $L : (\mathbb{R}^m, \|\cdot\|_2) \to (\mathbb{R}^n, \|\cdot\|_2)$.

18. Let $\mathcal{X}$ and $\mathcal{Y}$ be normed spaces and $L : \mathcal{X} \to \mathcal{Y}$ be a bounded linear operator. Prove the following:
    (a) $\|L(x)\| \le \|x\| \, \|L\|$;
    (b) $\|L\| = \sup_{\|x\|=1} \|L(x)\|$;
    (c) $\|L\| = \sup_{\|x\|\le 1} \|L(x)\|$.