55. Let $I \subset \mathbb{R}$ be an interval. A function $f : I \to \mathbb{R}$ is called convex if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y) \text{ whenever } x, y \in I \text{ and } \lambda \in [0,1].$$

(a) Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is convex.

(b) Assume f is differentiable. Show that f is convex iff f' is increasing.

(c) Show that $e : \mathbb{R} \to (0, \infty)$ and $-l : (0, \infty) \to \mathbb{R}$ are both convex.

56. Find the Taylor expansion of $f : (0,1] \to \mathbb{R}$ defined by $f(x) = l(1+x)$ at $x_0 = 0$.

57. Using only the definition of the Riemann integral, find $\int_a^b f(x) \mathrm{d}x$ with $0 \leq a < b$ in each of the following cases.

(a) $f(x) = x^2$;

(b) $f(x) = x^3$;

(c) $f(x) = 3x + 2x^2 - 5$.