Define
\[ c(t, x) = \pi N(d_+(T-t, x)) - Ke^{-r(T-t)}N(d_-(T-t, x)), \]

where
\[ d_\pm(\tau, x) = \frac{\ln \left( \frac{x}{K} \right) + \left( r \pm \frac{x^2}{2} \right) \tau}{\sigma \sqrt{\tau}}. \]

We have

(a) \[ d_+(\tau, x) - d_-(\tau, x) = \sigma \sqrt{\tau}; \]
(b) \[ N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}; \]
(c) \[ xN'(d_+(\tau, x)) - Ke^{-r\tau}N'(d_-(\tau, x)) = 0; \]
(d) \[ \frac{\partial d_+(T-t, x)}{\partial t} = \frac{\partial d_-(T-t, x)}{\partial t} - \frac{\sigma}{2\sqrt{T-t}}; \]
(e) \[ \frac{\partial c(t, x)}{\partial t} = -rKe^{-r(T-t)}N(d_-(T-t, x)) - \frac{\sigma x}{2\sqrt{T-t}} N'(d_+(T-t, x)); \]
(f) \[ \frac{\partial d_+(\tau, x)}{\partial x} = \frac{\partial d_-(\tau, x)}{\partial x} = \frac{1}{\sigma x \sqrt{\tau}}; \]
(g) \[ \frac{\partial c(t, x)}{\partial x} = N(d_+(T-t, x)); \]
(h) \[ \frac{\partial^2 c(t, x)}{\partial x^2} = \frac{1}{\sigma x \sqrt{T-t}} N'(d_+(T-t, x)); \]
(i) \[ \frac{\partial c(t, x)}{\partial t} + \frac{\partial^2 c(t, x)}{2 \partial x^2} + \frac{\partial^2 c(t, x)}{2} + r c(t, x) = 0; \]
(j) \[ \lim_{t \to T^-} c(t, x) = (x - K)^+; \]
(k) \[ c(t, x) \sim x - Ke^{-r(T-t)} \text{ for large } x; \]
(l) \[ c(t, x) \sim (x - Ke^{-r(T-t)})^+ \text{ for small } \sigma. \]

The price of a European call at time zero with expiration time \( T \) and strike price \( K \) is given by
\[ S(0)N(d_+(T, S(0))) - Ke^{-rT}N(d_-(T, S(0))). \]