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Existence of periodic solutions in predator–prey and competition dynamic systems☆

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Abstract

In this paper, we systematically explore the periodicity of some dynamic equations on time scales, which incorporate as special cases many population models (e.g., predator–prey systems and competition systems) in mathematical biology governed by differential equations and difference equations. Easily verifiable sufficient criteria are established for the existence of periodic solutions of such dynamic equations, which generalize many known results for continuous and discrete population models when the time scale $\mathbb{T}$ is chosen as $\mathbb{R}$ or $\mathbb{Z}$, respectively. The main approach is based on a continuation theorem in coincidence degree theory, which has been extensively applied in studying existence problems in differential equations and difference equations but rarely applied in dynamic equations on time scales. This study shows that it is unnecessary to explore the existence of periodic solutions of continuous and discrete population models in separate ways. One can unify such studies in the sense of dynamic equations on general time scales.

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1. Introduction

In the past decades, mathematical ecology has seen much progress, especially in population dynamics. Most natural environments are physically highly variable, and in response, birth rates, death rates, and other vital rates of populations, vary greatly in time. Theoretical evidence to date suggests that many population and community patterns represent intricate interactions between biology and variation in the physical environment (see [4] and other papers in the same issue). Therefore, the focus in theoretical models of population and community dynamics must be not only on how populations depend on their own population densities or the population densities of other organisms, but also on how

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populations change in response to the physical environment. When the environmental fluctuation is taken into account, a model must be nonautonomous, and hence, of course, more difficult to analyze in general. But, in doing so, one can and should also take advantage of the properties of those varying parameters. For example, one may assume the parameters are periodic or almost periodic for seasonal reasons. Due to the recognition that temporal fluctuations in the physical environment are a major driver of population fluctuations, there has been more and more theoretical attention to predict the characteristic of the resultant population fluctuations.

A very basic and important problem in the study of a population growth model with a periodic environment is the global existence and stability of a positive periodic solution, which plays a similar rôle as a globally stable equilibrium does in an autonomous model. Thus, it is reasonable to seek conditions under which the resulting periodic nonautonomous system would have a positive periodic solution that is globally asymptotically stable. Much progress has been seen in this direction. Careful investigation reveals that it is similar to explore the existence of periodic solutions for nonautonomous population models governed by ordinary differential equations and their discrete analogue in the approaches, the methods and the main results. For example, extensive research reveals that many results concerning the existence of periodic solutions of predator–prey systems modelled by differential equations can be carried over to their discrete analogues based on the coincidence theory, for example, [5,7,9–11,14,15,18,19]. It is natural to ask whether we can explore such an existence problem in a unified way.

The theory of calculus on time scales (see [2,3] and references cited therein) was initiated by Stefan Hilger in his Ph.D. Thesis in 1988 [13] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work. It has been created in order to unify the study of differential and difference equations. Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different from their continuous counterparts. The study of dynamic equations on general time scales can reveal such discrepancies and help avoid proving results twice—once for differential equations and once again for difference equations. The two main features of the calculus on time scales are unification and extension. To prove a result for a dynamic equation on a time scale is not only related to the set of real numbers or set of integers but those pertaining to more general time scales.

The principle aim of this paper is to systematically unify the existence of periodic solutions of population models modelled by ordinary differential equations and their discrete analogues in form of difference equations and to extend these results to more general time scales. The approach is based on a continuation theorem in coincidence degree, which has been widely applied to deal with the existence of periodic solutions of differential equations and difference equations. This paper is the first one to apply coincidence degree theory to explore the existence of periodic solutions of dynamic equations on time scales. The setup of this paper is as follows. In the coming section, we present some preliminary results such as the calculus on time scales and the continuation theorem in coincidence degree theory. Then we systematically explore the existence of periodic solutions of dynamic equations on time scales of predator–prey type and competition type. This study reveals that, when we deal with the existence of positive periodic solutions of population models, it is unnecessary to prove results for differential equations and separately again for difference equations. One can unify such problems in the frame of dynamic equations on time scales.

2. Preliminaries

In this section, we briefly give some elements of the time scales calculus, recall the continuation theorem from coincidence degree theory, and prove an auxiliary result that is needed in the paper.

First, let us present some foundational definitions and results from the calculus on time scales so that the paper is self-contained. For more details, one can see [2,3,13].

Notation 2.1. Throughout this paper, the symbol $\mathbb{T}$ denotes a *time scale*, i.e., an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Let $\omega > 0$. Throughout, the time scale $\mathbb{T}$ is assumed to be *ω-periodic*, i.e., $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$. In particular, the time scale $\mathbb{T}$ under consideration is unbounded above and below. Some examples of such time scales are

$$\mathbb{R}, \quad \mathbb{Z}, \quad \bigcup_{k\in\mathbb{Z}} [2k, 2k+1], \quad \bigcup_{k\in\mathbb{Z}} \bigcup_{n\in\mathbb{N}} \left\{ k + \frac{1}{n} \right\}.$$

Definition 2.1. We define the forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$, the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$, and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+ = [0, \infty)$ by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \quad \rho(t) := \sup\{s \in \mathbb{T} : s < t\} \quad \text{and} \quad \mu(t) = \sigma(t) - t \quad \text{for } t \in \mathbb{T},$$

respectively. If $\sigma(t) = t$, then t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise: left-scattered).

Definition 2.2. Assume $f : \mathbb{T} \to \mathbb{R}$ is a function and let $t \in \mathbb{T}$. Then we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t - \delta, t + \delta) \cap \mathbb{T}$ for some $\delta > 0$) such that

$$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \leqslant \varepsilon|\sigma(t) - s| \quad \text{for all } s \in U.$$

In this case, $f^{\Delta}(t)$ is called the delta (or Hilger) derivative of f at t. Moreover, f is said to be delta or Hilger differentiable on $\mathbb{T}$ if $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}$. A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$ for all $t \in \mathbb{T}$. Then we define

$$\int_r^s f(t)\Delta t = F(s) - F(r) \quad \text{for } r, s \in \mathbb{T}.$$

Definition 2.3. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{rd}(\mathbb{T})$.

Lemma 2.1. *Every rd-continuous function has an antiderivative.*

Lemma 2.2. *If* $a, b \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ *and* $f, g \in C_{rd}(\mathbb{T})$, *then*

(a) $\int_a^b [\alpha f(t) + \beta g(t)]\Delta t = \alpha \int_a^b f(t)\Delta t + \beta \int_a^b g(t)\Delta t$;
(b) *if* $f(t) \geqslant 0$ *for all* $a \leqslant t < b$, *then* $\int_a^b f(t)\Delta t \geqslant 0$;
(c) *if* $|f(t)| \leqslant g(t)$ *on* $[a, b) := \{t \in \mathbb{T} : a \leqslant t < b\}$, *then* $\left|\int_a^b f(t)\Delta t\right| \leqslant \int_a^b g(t)\Delta t$.

Notation 2.2. To facilitate the discussion below, we now introduce some notation to be used throughout this paper. Let

$$\kappa = \min\{[0, \infty) \cap \mathbb{T}\}, \quad I_\omega = [\kappa, \kappa + \omega] \cap \mathbb{T}, \quad g^u = \sup_{t \in \mathbb{T}} g(t), \quad g^l = \inf_{t \in \mathbb{T}} g(t),$$
$$\overline{g} = \frac{1}{\omega} \int_{I_\omega} g(s)\Delta s = \frac{1}{\omega} \int_\kappa^{\kappa+\omega} g(s)\Delta s,$$

where $g \in C_{rd}(\mathbb{T})$ is an ω-periodic real function, i.e., $g(t + \omega) = g(t)$ for all $t \in \mathbb{T}$.

Next, let us recall the continuation theorem in coincidence degree theory borrowing notations and terminology from [12], which will come into play later on.

Notation 2.3. Let X, Z be normed vector spaces, $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero and there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$, then it follows that $L|\operatorname{Dom} L \cap \operatorname{Ker} P : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 2.3 (*Continuation Theorem*). *Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Suppose*

(a) *For each $\lambda \in (0, 1)$, every solution z of $Lz = \lambda N z$ is such that $z \notin \partial\Omega$;*
(b) *$QNz \neq 0$ for each $z \in \partial\Omega \cap \operatorname{Ker} L$ and the Brouwer degree $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$.*

Then the operator equation $Lz = Nz$ has at least one solution lying in $\operatorname{Dom} L \cap \overline{\Omega}$.

In order to achieve the priori estimation in the case of dynamic equations on a time scale $\mathbb{T}$, we first prove the following inequalities, which will be very essential in this paper.

Lemma 2.4. *Let $t_1, t_2 \in I_\omega$ and $t \in \mathbb{T}$. If $g : \mathbb{T} \to \mathbb{R}$ is ω-periodic, then*

$$g(t) \leqslant g(t_1) + \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s \quad \text{and} \quad g(t) \geqslant g(t_2) - \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s.$$

Proof. We only show the first inequality as the proof of the second inequality is similar to the proof of the other one. Since g is ω-periodic, without any loss of generality, it suffices to show that the inequality is valid for $t \in I_\omega$. If $t = t_1$, then the first inequality is obviously true. If $t > t_1$, then one has

$$g(t) - g(t_1) \leqslant |g(t) - g(t_1)| = \left| \int_{t_1}^{t} g^\Delta(s)\Delta s \right| \leqslant \int_{t_1}^{t} |g^\Delta(s)|\Delta s \leqslant \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s,$$

and hence

$$g(t) \leqslant g(t_1) + \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s.$$

If $t < t_1$, then

$$g(t_1) - g(t) \geqslant -|g(t_1) - g(t)| = -\left| \int_{t}^{t_1} g^\Delta(s)\Delta s \right| \geqslant -\int_{t}^{t_1} |g^\Delta(s)|\Delta s \geqslant -\int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s,$$

that is

$$g(t) \leqslant g(t_1) + \int_\kappa^{\kappa+\omega} |g^\Delta(s)|\Delta s.$$

The proof is complete. □

Remark 2.1. If $\mathbb{T} = \mathbb{R}$, then the inequalities are standard for the Riemann integral, while if $\mathbb{T} = \mathbb{Z}$, then Lemma 2.4 reduces to the inequalities established by Fan and Wang [10, Lemma 3.2].

3. Predator–prey dynamic systems

The dynamic relationship between predators and their preys has long been and will continue to be one of the dominating themes in both ecology and mathematical biology due to its universal existence and importance [1]. Understanding the dynamical relationship between predator and prey is a central goal in ecology, and one significant component of the predator–prey relationship is the predator's rate of feeding upon prey, i.e., the so-called predator's functional response. In general, the functional response can be classified into two types: prey-dependent and predator-dependent. Prey dependent means that the functional response is only a function of the prey's density, while predator-dependent means that the functional response is a function of both the prey's and the predator's densities. Although the predator-dependent models that are considered fit those data reasonably well, no single functional response best describes all of the data sets. Theoretical studies have shown that the dynamics of models with predator-dependent functional responses can differ considerably from those with prey-dependent functional responses. Due to the fact that many results concerning the existence of periodic solutions of predator–prey systems modelled by differential equations can be carried over to their discrete analogues, in this section, we unify the existence of periodic solutions of predator–prey systems with different functional response in the framework of dynamic equations on time scales.

3.1. Predator–prey dynamic systems with Beddington–DeAngelis functional response

First, we focus on predator–prey systems with Beddington–DeAngelis functional response on time scales $\mathbb{T}$ of the form

$$\begin{cases} x^{\Delta}(t) = a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)\exp\{y(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}, \\ y^{\Delta}(t) = -d(t) + \dfrac{f(t)\exp\{x(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}, \end{cases} \tag{3.1}$$

where $a, b, c, d, f, \alpha, \beta, \gamma \in C_{\mathrm{rd}}(\mathbb{T})$ are ω-periodic such that

$$\overline{a}, \overline{d}, \gamma^l > 0 \quad \text{and} \quad b(t), c(t), f(t), \alpha(t), \beta(t) \geqslant 0 \quad \text{for all } t \in \mathbb{T}. \tag{3.2}$$

Remark 3.1. Let $\tilde{x}(t) = \exp\{x(t)\}$ and $\tilde{y}(t) = \exp\{y(t)\}$. If $\mathbb{T} = \mathbb{R}$, then (3.1) reduces to the standard predator–prey system with Beddington–DeAngelis functional response governed by the ordinary differential equations

$$\begin{cases} \tilde{x}'(t) = \tilde{x}(t)\left[a(t) - b(t)\tilde{x}(t) - \dfrac{c(t)\tilde{y}(t)}{\alpha(t)+\beta(t)\tilde{x}(t)+\gamma(t)\tilde{y}(t)}\right], \\ \tilde{y}'(t) = \tilde{y}(t)\left[-d(t) + \dfrac{f(t)\tilde{x}(t)}{\alpha(t)+\beta(t)\tilde{x}(t)+\gamma(t)\tilde{y}(t)}\right], \end{cases} \tag{3.3}$$

where $\tilde{x}(t)$ and $\tilde{y}(t)$ denote the density of the preys and the predators. The predator–prey systems of form (3.3) have been extensively studied [6]. If $\mathbb{T} = \mathbb{Z}$, then (3.1) is reformulated as

$$\begin{cases} \tilde{x}(t+1) = \tilde{x}(t)\exp\left[a(t) - b(t)\tilde{x}(t) - \dfrac{c(t)\tilde{y}(t)}{\alpha(t)+\beta(t)\tilde{x}(t)+\gamma(t)\tilde{y}(t)}\right], \\ \tilde{y}(t+1) = \tilde{y}(t)\exp\left[-d(t) + \dfrac{f(t)\tilde{x}(t)}{\alpha(t)+\beta(t)\tilde{x}(t)+\gamma(t)\tilde{y}(t)}\right], \end{cases} \tag{3.4}$$

which is the discrete time predator–prey system with Beddington–DeAngelis functional response and is also a discrete analogue of (3.3). Since (3.1) incorporates (3.3) and (3.4) as special cases, we call (3.1) the predator–prey dynamic system with Beddington–DeAngelis functional response on time scales.

In order to explore the existence of periodic solutions of (3.1), first we should embed our problem in the frame of coincidence degree theory. Define

$$\mathscr{L}^{\omega} = \{(u, v) \in C(\mathbb{T}, \mathbb{R}^2) : \ u(t+\omega) = u(t), \ v(t+\omega) = v(t) \text{ for all } t \in \mathbb{T}\},$$

$$\|(u, v)\| = \max_{t \in I_\omega} |u(t)| + \max_{t \in I_\omega} |v(t)| \quad \text{for } (u, v) \in \mathscr{L}^{\omega}.$$

It is not difficult to show that $\mathscr{L}^{\omega}$ is a Banach space when it is endowed with the above norm $\|\cdot\|$. Let

$$\mathscr{L}_0^{\omega} = \{(u, v) \in \mathscr{L}^{\omega} : \ \overline{u} = 0, \ \overline{v} = 0\},$$

$$\mathscr{L}_c^{\omega} = \{(u, v) \in \mathscr{L}^{\omega} : \ (u(t), v(t)) \equiv (h_1, h_2) \in \mathbb{R}^2 \text{ for } t \in \mathbb{T}\}.$$

Then it is easy to show that $\mathscr{L}_0^{\omega}$ and $\mathscr{L}_c^{\omega}$ are both closed linear subspaces of $\mathscr{L}^{\omega}$, $\mathscr{L}^{\omega} = \mathscr{L}_0^{\omega} \oplus \mathscr{L}_c^{\omega}$, and $\dim \mathscr{L}_c^{\omega} = 2$.

Theorem 3.1. *Assume* (3.2). *If*

$$\overline{a - c/\gamma} > 0 \quad \text{and} \quad (\overline{f} - \overline{d}\beta^u)(\overline{a - c/\gamma})\exp\{-(\overline{a + |a|})\omega\} - \overline{bd}\alpha^u > 0, \tag{3.5}$$

then (3.1) *has at least one* ω*-periodic solution.*

Proof. Let $X = Z = \mathscr{L}^{\omega}$ and define

$$N\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} = \begin{bmatrix} a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)\exp\{y(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}} \\ -d(t) + \dfrac{f(t)\exp\{x(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}} \end{bmatrix},$$

$$L\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^{\Delta} \\ y^{\Delta} \end{bmatrix}, \quad P\begin{bmatrix} x \\ y \end{bmatrix} = Q\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \overline{x} \\ \overline{y} \end{bmatrix}.$$

Then $\operatorname{Ker} L = \mathscr{L}_c^{\omega}$, $\operatorname{Im} L = \mathscr{L}_0^{\omega}$, and $\dim \operatorname{Ker} L = 2 = \operatorname{codim} \operatorname{Im} L$. Since $\mathscr{L}_0^{\omega}$ is closed in $\mathscr{L}^{\omega}$, it follows that L is a Fredholm mapping of index zero. It is not difficult to show that P and Q are continuous projections such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$. Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists and is given by

$$K_p\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} X - \overline{X} \\ Y - \overline{Y} \end{bmatrix} \quad \text{where } X(t) = \int_{\kappa}^{t} x(s)\Delta s \quad \text{and} \quad Y(t) = \int_{\kappa}^{t} y(s)\Delta s.$$

Thus

$$QN\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega}\left[a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)\exp\{y(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\right]\Delta t \\ \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega}\left[-d(t) + \dfrac{f(t)\exp\{x(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\right]\Delta t \end{bmatrix},$$

and

$$K_p(I-Q)N\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \int_{\kappa}^{t} N_1(s)\Delta s - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega}\int_{\kappa}^{t} N_1(s)\Delta s\Delta t - \left(t - \kappa - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega}(t-\kappa)\Delta t\right)\overline{N_1} \\ \int_{\kappa}^{t} N_2(s)\Delta s - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega}\int_{\kappa}^{t} N_2(s)\Delta s\Delta t - \left(t - \kappa - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega}(t-\kappa)\Delta t\right)\overline{N_2} \end{bmatrix}.$$

Obviously, QN and $K_p(I-Q)N$ are continuous. Since X is a Banach space, using the Arzelà–Ascoli theorem, it is easy to show that $\overline{K_p(I-Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Thus, N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we are in the position to search for an appropriate open, bounded subset Ω for the application of the continuation theorem, Lemma 2.3. For the operator equation $Lx = \lambda Nx$, $Ly = \lambda Ny$, $\lambda \in (0, 1)$, we have

$$\begin{cases} x^{\Delta}(t) = \lambda\left[a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)\exp\{y(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\right], \\ y^{\Delta}(t) = \lambda\left[-d(t) + \dfrac{f(t)\exp\{x(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\right]. \end{cases} \tag{3.6}$$

Assume that $(x, y) \in X$ is an arbitrary solution of system (3.6) for a certain $\lambda \in (0, 1)$. Integrating both sides of (3.6) over the interval $[\kappa, \kappa+\omega]$, we obtain

$$\begin{cases} \overline{a}\omega = \int_{\kappa}^{\kappa+\omega}\left[b(t)\exp\{x(t)\} + \dfrac{c(t)\exp\{y(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\right]\Delta t, \\ \overline{d}\omega = \int_{\kappa}^{\kappa+\omega}\left[\dfrac{f(t)\exp\{x(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\right]\Delta t. \end{cases} \tag{3.7}$$

From (3.6) and (3.7), we obtain

$$\begin{aligned}\int_\kappa^{\kappa+\omega}|x^\Delta(t)|\Delta t &\leqslant \lambda\Bigg[\int_\kappa^{\kappa+\omega}|a(t)|\Delta t+\int_\kappa^{\kappa+\omega}b(t)\exp\{x(t)\}\Delta t \\ &\quad+\int_\kappa^{\kappa+\omega}\frac{c(t)\exp\{y(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\Delta t\Bigg] \\ &=\lambda(\overline{a+|a|})\omega<(\overline{a+|a|})\omega,\end{aligned}$$

$$\begin{aligned}\int_\kappa^{\kappa+\omega}|y^\Delta(t)|\Delta t &\leqslant \lambda\Bigg[\int_\kappa^{\kappa+\omega}|d(t)|\Delta t+\int_\kappa^{\kappa+\omega}\frac{f(t)\exp\{x(t)\}}{\alpha(t)+\beta(t)\exp\{x(t)\}+\gamma(t)\exp\{y(t)\}}\Delta t\Bigg] \\ &=\lambda(\overline{d+|d|})\omega<(\overline{d+|d|})\omega.\end{aligned}$$

Note that since $(x, y) \in X$, there exist $\xi_i, \eta_i \in [\kappa, \kappa+\omega]$, $i \in \{1, 2\}$, such that

$$\begin{aligned}&x(\xi_1)=\min_{t\in[\kappa,\kappa+\omega]}x(t), \quad x(\eta_1)=\max_{t\in[\kappa,\kappa+\omega]}x(t),\\ &y(\xi_2)=\min_{t\in[\kappa,\kappa+\omega]}y(t), \quad y(\eta_2)=\max_{t\in[\kappa,\kappa+\omega]}y(t).\end{aligned} \tag{3.8}$$

From (3.8) and the first equation of (3.7), we have

$$\bar{a}\omega\leqslant\int_\kappa^{\kappa+\omega}\left[b(t)\exp\{x(\eta_1)\}+\frac{c(t)}{\gamma(t)}\right]\Delta t=\bar{b}\omega\exp\{x(\eta_1)\}+\overline{(c/\gamma)}\omega.$$

By the first part of the assumption in (3.5), we can conclude that

$\bar{b} > 0$ necessarily must hold.

Then $x(\eta_1) \geqslant \ln\{(\overline{a - c/\gamma})/\bar{b}\} =: l_1$, and therefore, using the second inequality in Lemma 2.4,

$$x(t)\geqslant x(\eta_1)-\int_\kappa^{\kappa+\omega}|x^\Delta(t)|\Delta t>l_1-(\overline{a+|a|})\omega=:H_2. \tag{3.9}$$

On the other hand, from (3.8) and the first equation of (3.7), we also obtain

$$\bar{a}\omega\geqslant\int_\kappa^{\kappa+\omega}b(t)\exp\{x(\xi_1)\}\Delta t=\bar{b}\omega\exp\{x(\xi_1)\},$$

which reduces to $x(\xi_1) \leqslant \ln\{\bar{a}/\bar{b}\} =: L_1$, and hence, using the first inequality in Lemma 2.4,

$$x(t)\leqslant x(\xi_1)+\int_\kappa^{\kappa+\omega}|x^\Delta(t)|\Delta t<L_1+(\overline{a+|a|})\omega=:H_1,$$

which, together with (3.9), leads to $\max_{t\in[\kappa,\kappa+\omega]}|x(t)| \leqslant \max\{|H_1|, |H_2|\} =: B_1$. From (3.8) and the second equation of (3.7), we can derive that

$$\bar{d}\omega\leqslant\int_\kappa^{\kappa+\omega}\frac{f(t)\exp\{x(t)\}}{\beta^l\exp\{x(t)\}+\gamma^l\exp\{y(t)\}}\Delta t\leqslant\int_\kappa^{\kappa+\omega}\frac{f(t)e^{H_1}}{\beta^l e^{H_1}+\gamma^l\exp\{y(\xi_2)\}}\Delta t=\frac{\omega\bar{f}e^{H_1}}{\beta^l e^{H_1}+\gamma^l\exp\{y(\xi_2)\}},$$

so $\exp\{y(\xi_2)\} \leqslant ((\bar{f} - \bar{d}\beta^l)e^{H_1})/\bar{d}\gamma^l$. Thus

$\bar{f} - \bar{d}\beta^l > 0$ necessarily must hold.

Then $y(\xi_2) \leqslant \ln\{(\bar{f} - \bar{d}\beta^l)e^{H_1}/\bar{d}\gamma^l\} =: L_2$. Hence, by using the first inequality in Lemma 2.4,

$$y(t)\leqslant y(\xi_2)+\int_\kappa^{\kappa+\omega}|y^\Delta(t)|\Delta t<L_2+(\overline{d+|d|})\omega=:H_3. \tag{3.10}$$

We can also derive from the second equation of (3.7) that

$$\overline{d}\omega \geqslant \int_{\kappa}^{\kappa+\omega} \frac{f(t)\exp\{x(t)\}}{\alpha^u + \beta^u \exp\{x(t)\} + \gamma^u \exp\{y(\eta_2)\}}\Delta t \geqslant \int_{\kappa}^{\kappa+\omega} \frac{f(t)e^{H_2}}{\alpha^u + \beta^u e^{H_2} + \gamma^u \exp\{y(\eta_2)\}}\Delta t.$$

Then it follows that

$$\exp\{y(\eta_2)\} \geqslant \frac{(\overline{f} - \overline{d}\beta^u)((\overline{a - c/\gamma})/\overline{b})\exp\{-(\overline{a + |a|})\omega\} - \overline{d}\alpha^u}{\overline{d}\gamma^u} =: l_2^*.$$

By the second part of the assumption in (3.5), we can conclude that $l_2^* > 0$ so that $y(\eta_2) \geqslant \ln(l_2^*) =: l_2$ and hence, by using the second inequality in Lemma 2.4,

$$y(t) \geqslant y(\eta_2) - \int_{\kappa}^{\kappa+\omega} |y^{\Delta}(t)|\Delta t > l_2 - \omega(\overline{d + |d|}) =: H_4,$$

which, together with (3.10), leads to $\max_{t\in[\kappa,\kappa+\omega]} |y(t)| \leqslant \max\{|H_3|, |H_4|\} =: B_2$. Obviously, B_1 and B_2 are both independent of λ. Let $B = B_1 + B_2 + B_3$, where $B_3 > 0$ is taken sufficiently large such that $B_3 \geqslant |l_1| + |L_1| + |l_2| + |L_2|$.

Next let us consider the algebraic equations

$$\begin{cases} \overline{a} - \overline{b}\exp\{x\} - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega} \dfrac{vc(t)\exp\{y\}}{\alpha(t) + \beta(t)\exp\{x\} + \gamma(t)\exp\{y\}}\Delta t = 0, \\ -\overline{d} + \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega} \dfrac{f(t)\exp\{x\}}{\alpha(t) + \beta(t)\exp\{x\} + \gamma(t)\exp\{y\}}\Delta t = 0 \end{cases} \tag{3.11}$$

for $(x, y) \in \mathbb{R}^2$, where $v \in [0, 1]$ is a parameter. By carrying out similar arguments as above, it is not difficult to show that any solution (x^*, y^*) of (3.11) with $v \in [0, 1]$ satisfies

$$l_1 \leqslant x^* \leqslant L_1 \quad \text{and} \quad l_2 \leqslant y^* \leqslant L_2. \tag{3.12}$$

Now we define $\Omega = \{(x, y) \in X : \|(x, y)\| < B\}$. Then it is clear that Ω satisfies the requirement (a) of Lemma 2.3. If $(x, y) \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap \mathbb{R}^2$, then (x, y) is a constant vector in $\mathbb{R}^2$ with $\|(x, y)\| = |x| + |y| = B$. Then from (3.12) and the definition of B, we have

$$QN\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \overline{a} - \overline{b}\exp\{x\} - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega} \dfrac{c(t)\exp\{y\}}{\alpha(t) + \beta(t)\exp\{x\} + \gamma(t)\exp\{y\}}\Delta t \\ -\overline{d} + \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega} \dfrac{f(t)\exp\{x\}}{\alpha(t) + \beta(t)\exp\{x\} + \gamma(t)\exp\{y\}}\Delta t \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Moreover, note that $J = I$ since $\operatorname{Im} Q = \operatorname{Ker} L$. In order to compute the Brouwer degree, let us consider the homotopy

$$H_v(x, y) = vQN(x, y) + (1 - v)G(x, y) \quad \text{for } v \in [0, 1],$$

where

$$G(x, y) = \begin{bmatrix} \overline{a} - \overline{b}\exp\{x\} \\ \overline{d} - \dfrac{1}{\omega}\int_{\kappa}^{\kappa+\omega} \dfrac{f(t)\exp\{x\}}{\alpha(t) + \beta(t)\exp\{x\} + \gamma(t)\exp\{y\}}\Delta t \end{bmatrix}.$$

From (3.12), it is easy to show that $0 \notin H_v(\partial\Omega \cap \operatorname{Ker} L)$ for $v \in [0, 1]$. Moreover, one can easily show that the algebraic equation $G(x, y) = 0$ has a unique solution in $\mathbb{R}^2$. By the invariance property of homotopy, direct calculation produces

$$\deg(JQN, \Omega \cap \operatorname{Ker} L, 0) = \deg(QN, \Omega \cap \operatorname{Ker} L, 0) = \deg(G, \Omega \cap \operatorname{Ker} L, 0) \neq 0,$$

where $\deg(\cdot, \cdot, \cdot)$ is the Brouwer degree. By now we have proved that Ω satisfies all requirements of Lemma 2.3. Thus $Lz = Nz$ has at least one solution in $\operatorname{Dom} L \cap \overline{\Omega}$, i.e., (3.1) has at least one ω-periodic solution in $\operatorname{Dom} L \cap \overline{\Omega}$. The proof is complete. □

Remark 3.2. If $\mathbb{T} = \mathbb{R}$, then (3.1) is the continuous predator–prey system with Beddington–DeAngelis functional response and Theorem 3.1 is [6, Theorem 3.2].

Remark 3.3. If $\alpha(t) \equiv 0$, then (3.1) reduces to the ratio-dependent predator–prey system. If in addition $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$, then (3.1) reduces to the continuous or the discrete ratio-dependent predator–prey system, and Theorem 3.1 unifies and generalizes the main results in [9,10].

Example 3.1. Consider two insect populations (one of them the predator, the other one the prey) that are both continuous while in season (say during the six warm months of the year), die out in (say) winter, while their eggs are incubating or dormant, and then both hatch in a new season, both of them giving rise to nonoverlapping populations. This situation can be modelled using the time scale

$$\mathbb{T} = \bigcup_{k \in \mathbb{Z}} [2k, 2k+1] \quad \text{with } \omega = 1.$$

If the model assumes a Beddington–DeAngelis functional response as in (3.1), and if the assumptions (3.2) and (3.5) hold for the underlying parameters, then, by Theorem 3.1, there exists a 1-periodic solution of (3.1).

3.2. Predator–prey dynamic systems with Holling-type functional response

Consider the predator–prey system on time scales with Holling-type functional response

$$\begin{cases} x^{\Delta}(t) = a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)\exp\{y(t)\}}{1 + m(t)\exp\{x(t)\}}, \\ y^{\Delta}(t) = -d(t) + \dfrac{f(t)\exp\{x(t)\}}{1 + m(t)\exp\{x(t)\}} \end{cases} \tag{3.13}$$

and

$$\begin{cases} x^{\Delta}(t) = a(t) - b(t)\exp\{x(t)\} - \dfrac{c(t)\exp\{y(t) + x(t)\}}{1 + m(t)\exp\{2x(t)\}}, \\ y^{\Delta}(t) = -d(t) + \dfrac{f(t)\exp\{2x(t)\}}{1 + m(t)\exp\{2x(t)\}}, \end{cases} \tag{3.14}$$

where $a, b, c, d, m, f \in C_{\rm rd}(\mathbb{T}, \mathbb{R})$ are nonnegative ω-periodic.

Remark 3.4. In systems (3.13) and (3.14), if $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$, then (3.13) and (3.14) reduce to a continuous or discrete predator–prey systems with Holling-type II or III functional responses, which have been extensively studied in the literature [15,19].

Following a similar strategy as in [8,20], one can reach the following two conclusions. Since we supplied the details for systems with Beddington–DeAngelis functional response in the time scales case and since the proofs of the following two theorems are similar to those in [8,20], the details are omitted here.

Theorem 3.2. *If* $\overline{f} > m^{v}\overline{d}$ *and* $\overline{a} > \overline{bd}/(\overline{f} - m^{v}\overline{d})\, \exp\{2\overline{a}\omega\}$, *then system* (3.13) *has at least one* ω*-periodic solution.*

Theorem 3.3. *If* $\overline{f} > m^{v}\overline{d}$ *and* $\overline{a} > \overline{b}[\overline{d}/(\overline{f} - m^{v}\overline{d})]^{1/2} \exp\{2\overline{a}\omega\}$, *then system* (3.14) *has at least one* ω*-periodic solution.*

3.3. Semi-ratio-dependent predator–prey dynamic systems

Consider the nonautonomous semi-ratio-dependent predator–prey system on time scales

$$\begin{cases} x^{\Delta}(t) = a(t) - b(t)\exp\{x(t)\} - c(t, \exp\{x(t)\})\exp\{y(t) - x(t)\}, \\ y^{\Delta}(t) = d(t) - e(t)\exp\{y(t) - x(t)\}, \end{cases} \tag{3.15}$$

where $c(t, x)$ is a prey-dependent functional response, which can be either of the following forms (in order, we call them the functional response of type 1–5)

$$m(t)x; \quad \frac{m(t)x}{A+x}; \quad \frac{m(t)x^n}{A+x^n}, \quad n \geqslant 2; \quad \frac{m(t)x^2}{(A+x)(B+x)}; \quad m(t)(1-\exp\{-Ax\}).$$

In (3.15), we consider the following assumptions.

(H_1) $a, b, d, e \in C_{rd}(\mathbb{T}, \mathbb{R}^+)$ are nonnegative ω-periodic;
(H_2) $c : \mathbb{T} \times \mathbb{R}^+ \to \mathbb{R}^+$ is rd-continuous and ω-periodic with respect to the first variable, and is differentiable with respect to the second variable, and $(\partial c/\partial x)(t, x) > 0$ for any $t \in \mathbb{T}$, and $(\partial c/\partial x)(t, x)$ is bounded with respect to t;
(H_3) there exists an ω-periodic function $C_0 \in C_{rd}(\mathbb{T}, \mathbb{R}^+)$ with $c(t, x) \leqslant C_0(t)x$ for any $t \in \mathbb{T}$;
(H_4) there exists an ω-periodic function $C_1 \in C_{rd}(\mathbb{T}, \mathbb{R}^+)$ with $c(t, x) \leqslant C_1(t)$ for any $t \in \mathbb{T}$.

By carrying out similar arguments as in [11,18], we present, without including the proofs, the following two results.

Theorem 3.4. *Assume that* (H_1), (H_2), *and* (H_3) *hold. If* $\overline{be} > \overline{C_0 d}\, \exp\{(\overline{a + |a| + d + |d|})\omega\}$, *then* (3.15) *has at least one* ω*-periodic solution.*

Theorem 3.5. *Assume that* (H_1), (H_2), *and* (H_4) *hold. If* $e^l\overline{a} > C_1^u\overline{d}$, *then* (3.15) *has at least one* ω*-periodic solution.*

Remark 3.5. If $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$, then (3.15) reduces to the continuous or discrete semi-ratio predator–prey system investigated in [14,18] or [11], respectively. Theorems 3.4 and 3.5 unify and generalize the main results in [18] and [11].

4. Competition dynamic systems

Competition poses an important rôle in mathematical ecology, and it has been studied extensively. In this section, we outline how to conclude the existence of periodic solutions of competition dynamic systems on time scales.

Consider the generalized n-species Gilpin–Ayala competition system with several deviating arguments on time scales

$$y_i^{\Delta}(t) = r_i(t) - \sum_{j=1}^{n} a_{ij}(t) \exp\{\theta_{ij} y_j(t)\} \quad \text{for } i \in \{1, 2, \ldots, n\}, \tag{4.1}$$

where $r_i, a_{ij} \in C_{rd}(\mathbb{T}, \mathbb{R}^+)$, $i, j \in \{1, 2, \ldots, n\}$, are ω-periodic and bounded above and below by positive constants.

Remark 4.1. If $\mathbb{T}=\mathbb{R}$ or $\mathbb{T}=\mathbb{Z}$ and $\tilde{y}_i(t)=\exp\{y_i(t)\}$, then (4.1) reduces to the continuous or discrete time Gilpin–Ayala competition system with several deviating arguments

$$\tilde{y}_i'(t) = \tilde{y}_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\tilde{y}_j(t))^{\theta_{ij}}\right] \quad \text{for } i \in \{1, 2, \ldots, n\}, \quad t \in \mathbb{R}$$

or

$$\tilde{y}_i(t+1) = \tilde{y}_i(t) \exp\left\{r_i(t) - \sum_{j=1}^{n} a_{ij}(t)(\tilde{y}_j(t))^{\theta_{ij}}\right\} \quad \text{for } i \in \{1, 2, \ldots, n\}, \quad t \in \mathbb{Z}.$$

Therefore, we might as well recognize system (4.1) as a generalized n-species Gilpin–Ayala competition dynamic system with several deviating arguments on a time scale. The model (4.1) is very general and includes many ecological models as special cases on time scales, e.g., if $n = 1$ and $\theta_{ij} \equiv 1$, then (4.1) is a logistic equation on time scales; if $\theta_{ij} \equiv 1$, then (4.1) is the classical n-species Lotka–Volterra competition system with periodic coefficients on time scales; if $\theta_{ij} \equiv 1$ for $i \neq j$, then (4.1) is the classical Gilpin–Ayala competition model with periodic environment on time scales.

To prove Theorem 4.1, we proceed similarly as in the proof of Theorem 3.1, where the spaces $\mathscr{L}^\omega$ etc. now take the following forms:

$$\mathscr{L}^\omega = \{y = (y_1, y_2, \ldots, y_n) \in C(\mathbb{T}, \mathbb{R}^n) : \ y(t+\omega) = y(t) \text{ for all } t \in \mathbb{T}\},$$

$$\|y\| = \left\{ \sum_{i=1}^{n} \left(\max_{t \in I_\omega} |y_i(t)| \right)^2 \right\}^{1/2} \quad \text{for } y \in \mathscr{L}^\omega.$$

It is not difficult to show that $\mathscr{L}^\omega$ is a Banach space when it is endowed with the above norm $\|\cdot\|$. Let

$$\mathscr{L}_0^\omega = \{y \in \mathscr{L}^\omega : \overline{y} = 0\}, \quad \mathscr{L}_c^\omega = \{y \in \mathscr{L}^\omega : y(t) \equiv h \in \mathbb{R}^n \text{ for } t \in \mathbb{T}\}.$$

Then it is easy to show that $\mathscr{L}_0^\omega$ and $\mathscr{L}_c^\omega$ are both closed linear subspaces of $\mathscr{L}^\omega$, $\mathscr{L}^\omega = \mathscr{L}_0^\omega \oplus \mathscr{L}_c^\omega$, and $\dim \mathscr{L}_c^\omega = n$. With these alterations compared to the proof of Theorem 3.1, the proof of our last result is very similar to the proof of [7, Theorem 2.1].

Theorem 4.1. *If the system of algebraic equations*

$$g(u) = \left(\overline{r}_i - \sum_{j=1}^{n} \overline{a}_{ij} u_j^{\theta_{ij}} \right)_{n \times 1} = 0$$

has finitely many solutions $u^* = (u_1^*, \ldots, u_n^*) \in \mathbb{R}_+^n$ *with* $u_i^* > 0$ *and* $\sum_{u^*} \operatorname{sgn} J_g(u^*) \neq 0$, *and if*

$$r_i > \sum_{j=1, j \neq i}^{n} \overline{a}_{ij} \left(\frac{\overline{r}_j}{\overline{a}_{jj}} \right)^{\theta_{ij}/\theta_{jj}} \exp\{\theta_{ij} 2 \overline{r}_j \omega\},$$

then system (4.1) *has at least one* ω*-periodic solution.*

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