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Exponential synchronization of chaotic neural networks with mixed delays and impulsive effects via output coupling with delay feedback

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ABSTRACT

In this paper, we deal with the exponential synchronization problem for a class of chaotic neural networks with mixed delays and impulsive effects via output coupling with delay feedback. The mixed delays in this paper include time-varying delays and unbounded distributed delays. By using a Lyapunov–Krasovskiĭ functional, a drive–response concept and a linear matrix inequality (LMI) approach, several sufficient conditions are established that guarantee the exponential synchronization of the neural networks. Also, the estimation gains can be easily obtained. Finally, a numerical example and its simulation are given to show the effectiveness of the obtained results.

1. Introduction

In the past several years, control and synchronization problems of chaotic systems have been extensively studied, due to their potential applications in many areas [1–12]. For instance, they are used to understand self-organizational behavior in the brain [4] as well as in ecological systems [5], and they also have been applied to produce secure message communication between a sender and a receiver [6–8]. It is worth mentioning that, since Aihara et al. [13] first introduced chaotic neural network models in order to simulate the chaotic behavior of biological neurons, chaotic neural networks have been successfully applied in combinational optimization, parallel recognition, secure communication, and other areas [14,15]. Actually, chaotic neural networks as special complex networks can exhibit some complicated dynamics and even chaotic behaviors [16,17]. Therefore the investigation of synchronization of chaotic neural networks is of practical importance, and many interesting results have been obtained via different approaches in recent years. For instance, one can consult Cao et al. [18–22], Masoller [23], Cheng et al. [24–26], and the references therein.

Distributed delays including bounded and unbounded distributed delays in neural networks [27] have recently been studied extensively. In fact, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths, and hence there is a distribution of conduction velocities along these pathways and a distribution of propagation delays. Nowadays, there are many results dealing with stability and periodic oscillation for neural networks with distributed delays; see [28,29] and the references therein. Some authors have also considered chaos synchronization phenomena in neural networks with distributed delays; see [30–34]. However, there are some disadvantages of those existing chaos synchronization conditions including the following.

(i) Most of the above studies are valid only for chaotic neural networks with bounded distributed delays. They cannot be applied to chaotic neural networks with unbounded distributed delays.

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(ii) These existing synchronization conditions are mainly obtained via state-feedback controller or delayed feedback controller. In fact, in many real networks, only output signals can be measured [35,36]. Synchronization via output coupling is rarely considered in the literature.
(iii) In [20,22,24,26,30–32,37–40], all of the given results require that the time delays are constant delays or time-varying delays that are differentiable such that their derivatives are not greater than one or finite. On the other hand, neural networks are often subject to impulsive perturbations that in turn affect dynamical behavior of the systems [38]. Hence, it is important to consider both distributed delays and impulsive effects when investigating the dynamics of neural networks.

So far, several interesting results have been presented that have focused on the chaos synchronization of neural networks with discrete delays and impulsive effects, see [22,39]. However, there are very few results on the chaos synchronization issue for neural networks with distributed delays and impulsive effects [40], especially on neural networks with unbounded distributed delays. In this case, it is interesting and challenging to further study the chaos synchronization of neural networks with unbounded distributed delays and impulsive effects.

This paper, inspired by the above works, addresses the synchronization problem of chaotic neural networks with mixed delays and impulsive effects. The mixed delays include time-varying delays and unbounded distributed delays. Utilizing the drive–response concept, a control law associated with output coupling with delay feedback is derived in order to achieve exponential synchronization of two identical chaotic neural networks. Based on a Lyapunov–Krasovskiĭ functional and a linear matrix inequality (LMI) approach, several LMI-based conditions are obtained that guarantee chaos synchronization. Moreover, it is very convenient to apply these LMI-based conditions to real networks.

This paper is organized as follows. In Section 2, we introduce some basic definitions and notation. In Section 3, the main results for exponential synchronization of chaotic neural networks are presented. A numerical example is given in Section 4 in order to demonstrate the effectiveness of our results. Finally, in Section 5, some conclusions are summarized.

2. Notations and preliminaries

Let $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{R}^n$ denote the sets of real numbers, positive integers, and n-dimensional real vectors equipped with the Euclidean norm $\|\cdot\|$, respectively. Let A be a matrix. The notation $A > 0$ ($A < 0$) means that A is symmetric and positive (negative) definite. Moreover, A^T and A^{-1} denote the transpose and the inverse of A, respectively. We also put $\Lambda = \{1, 2, \dots, n\}$. Next, the notation $\star$ always denotes the symmetric block in a symmetric matrix.

The set $\mathrm{PC}_{\mathrm{b}}^1$ is defined to be the set of all functions $\psi : (-\infty, 0] \to \mathbb{R}^n$ such that ψ is continuously differentiable and bounded everywhere except at a finite number of points t, at which $\psi(t^+)$, $\psi(t^-)$, $\dot{\psi}(t^+)$ and $\dot{\psi}(t^-)$ exist, $\psi(t^+) = \psi(t)$, $\dot{\psi}(t^+) = \dot{\psi}(t)$, where $\dot{\psi}$ denotes the derivative of ψ. For any $\psi \in \mathrm{PC}_{\mathrm{b}}^1$ and $\kappa < 0$, we introduce the norm

$$\|\psi\|_\kappa = \sqrt{\max\left\{\max_{\theta\le 0}\sum_{i=1}^n |\psi_i^2(\theta)|, \max_{-\kappa\le\theta\le 0}\sum_{i=1}^n |\dot{\psi}_i^2(\theta)|\right\}}.$$

In this paper, we consider the model

$$\begin{cases} \dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + W\displaystyle\int_{-\infty}^t h(t-s)f(x(s))\mathrm{d}s + I(t), \quad t \neq t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = -D_k x(t_k^-), \quad k \in \mathbb{N}, \\ x(s) = \phi(s), \quad s \in (-\infty, 0], \end{cases} \tag{2.1}$$

where the *impulse times* t_k satisfy

$$0 = t_0 < t_1 < \cdots < t_k < \cdots, \quad \lim_{k\to\infty} t_k = \infty;$$

$x = (x_1, \dots, x_n)^T$ is the *neuron state vector* of the neural network; $C > 0$ is a diagonal matrix; A, B, W are the *connection weight matrix*, the *delayed weight matrix* and the *distributively delayed connection weight matrix*, respectively; I is a time-varying input vector; τ is a time-varying *transmission delay* of the neural network; $f(x) = (f_1(x_1), \dots, f_n(x_n))^T$ represents the *neuron activation function*; $h = \mathrm{diag}(h_1, \dots, h_n)$ is the *delay kernel function*; D_k is a diagonal matrix, called the *impulsive matrix*; $\phi \in \mathrm{P}$, where P is an open set in $\mathrm{PC}_{\mathrm{b}}^1$.

Throughout this paper, the following assumptions are employed:

(H_1) The neuron activation functions $f_j, j \in \Lambda$, are bounded and satisfy

$$\sigma_j^- \le \frac{f_j(u) - f_j(v)}{u - v} \le \sigma_j^+, \quad j \in \Lambda$$

for any $u, v \in \mathbb{R}, u \neq v$, where $\sigma_j^-, \sigma_j^+, j \in \Lambda$, are some real constants which may be positive, zero or negative.

(H$_2$) The delay kernels $h_j, j \in \Lambda$, are some real-valued nonnegative continuous functions defined in $[0, \infty)$ satisfying

$$\widetilde{h}_j := \int_0^\infty h_j(s)\mathrm{d}s > 0 \quad \text{and} \quad \widehat{h}_j := \int_0^\infty h_j(s)\mathrm{e}^{\eta s}\mathrm{d}s < \infty, \quad j \in \Lambda$$

for some $\eta > 0$.

(H$_3$) The transmission delay τ is time-varying and satisfies $0 \le \tau(t) \le \widetilde{\tau}$ for some $\widetilde{\tau} > 0$.

Remark 2.1. For the synchronization conditions given in [20,22,24,26,30–32,37–40], the time delays are constant delays or time-varying delays that are differentiable such that their derivatives are not greater than one or finite. Note that in our assumption (H$_3$) we do not impose those restrictions on our time-varying delays, which means that our presented results have wider adaptive range.

Based on the drive–response concept for synchronizatiog of coupled chaotic systems [1], the corresponding response system of (2.1) is given by

$$\begin{cases} \dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t-\tau(t))) + W\displaystyle\int_{-\infty}^{t} h(t-s)f(y(s))\mathrm{d}s + I(t) + u(t), & t \neq t_k, \\ \Delta y(t_k) = y(t_k) - y(t_k^-) = -D_k y(t_k^-), & k \in \mathbb{N}, \\ y(s) = \varphi(s), & s \in (-\infty, 0], \end{cases} \tag{2.2}$$

where $\varphi \in \mathrm{P}$; C, A, B and W are matrices which are the same as in the model (2.1); u is the controller.

Define the *synchronization error signal* as $e = (e_1, \dots, e_n)^T := y - x$. Then the error dynamical system between (2.1) and (2.2) is

$$\begin{cases} \dot{e}(t) = -Ce(t) + Ag(e(t)) + Bg(e(t-\tau(t))) + W\displaystyle\int_{-\infty}^{t} h(t-s)g(e(s))\mathrm{d}s + u(t), & t \neq t_k, \\ \Delta e(t_k) = e(t_k) - e(t_k^-) = -D_k e(t_k^-), & k \in \mathbb{N}, \\ e(s) = \varphi(s) - \phi(s), & s \in (-\infty, 0], \end{cases}$$

where $g(e(\cdot)) = f(e(\cdot) + x(\cdot)) - f(x(\cdot))$.

The control input in the response system is designed as

$$u(t) = K_1 \left[f(y(t)) - f(x(t))\right] + K_2 \left[f(y(t-\tau(t))) - f(x(t-\tau(t)))\right], \tag{2.3}$$

where K_1 and K_2 are the gain matrices to be determined. With the control law (2.3) and the notation

$$\widetilde{A} = A + K_1 \quad \text{and} \quad \widetilde{B} = B + K_2,$$

the error dynamics can be expressed by

$$\begin{cases} \dot{e}(t) = -Ce(t) + \widetilde{A}g(e(t)) + \widetilde{B}g(e(t-\tau(t))) + W\displaystyle\int_{-\infty}^{t} h(t-s)g(e(s))\mathrm{d}s, & t \neq t_k, \\ \Delta e(t_k) = e(t_k) - e(t_k^-) = -D_k e(t_k^-), & k \in \mathbb{N}, \\ e(s) = \varphi(s) - \phi(s), & s \in (-\infty, 0]. \end{cases} \tag{2.4}$$

Remark 2.2. It is well known that time delays always influence the dynamic properties of chaotic delayed neural networks, which may cause periodic oscillations, bifurcation and chaotic attractors and so on. Moreover, in many real networks, only output signals can be measured. Hence it is important to consider the time-delay feedback control $K_2\left[f(y(t-\tau(t))) - f(x(t-\tau(t)))\right]$ in the response system. We refer to this as output coupling with delay feedback.

Finally, we define

$$\Sigma_1 = \mathrm{diag}(\sigma_1^-\sigma_1^+, \dots, \sigma_n^-\sigma_n^+) \quad \text{and} \quad \Sigma_2 = \mathrm{diag}\left(\frac{\sigma_1^- + \sigma_1^+}{2}, \dots, \frac{\sigma_n^- + \sigma_n^+}{2}\right).$$

Definition 2.3. The models (2.1) and (2.2) are said to be *exponentially synchronized* if there exist constants $\lambda > 0$ and $M \ge 1$ such that $\|e(t)\| \le M\,\|\varphi - \phi\|_{\widetilde{\tau}}\,\mathrm{e}^{-\lambda t}$ for all $t > 0$. Here, λ is called the *convergence rate* (or degree) of exponential synchronization.

3. Criteria for synchronization

In this section, we investigate exponential synchronization of the models (2.1) and (2.2) by constructing suitable Lyapunov–Krasovskiĭ functionals.

Theorem 3.1. *Assume* (H$_1$)–(H$_3$). *Then the models* (2.1) *and* (2.2) *are exponentially synchronized if there exist five constants* $\alpha \in (0, \eta)$, $\beta, \gamma > 0$, $\delta \in [0, \alpha)$, $\widetilde{M} > 1$, *an* $n \times n$*-matrix* $P > 0$, *an* $n \times n$*-matrix* Q_1, *three diagonal positive definite*

$n \times n$-matrices $Q_2 = \mathrm{diag}(q_1^{(2)}, \dots, q_n^{(2)})$, U_1, U_2, and a $2n \times 2n$-matrix $T := \begin{pmatrix} T_{11} & T_{12} \\ \star & T_{22} \end{pmatrix} > 0$ such that

$$\Psi := \begin{pmatrix} \Psi_{11} & \Psi_{12} & \gamma T_{12}^T & \Psi_{14} & Q_1\widetilde{B} & Q_1 W \\ \star & \Psi_{22} & 0 & \beta Q_1 \widetilde{A} & \beta Q_1 \widetilde{B} & \beta Q_1 W \\ \star & \star & \Psi_{33} & 0 & U_2 \Sigma_2 & 0 \\ \star & \star & \star & Q_2 H - U_1 & 0 & 0 \\ \star & \star & \star & \star & -U_2 & 0 \\ \star & \star & \star & \star & \star & -Q_2 \end{pmatrix} < 0 \tag{3.1}$$

and

$$\prod_{k=1}^{m} \max\left\{ \frac{\mu_k}{\mu_{\min}},\ 1 \right\} \le \widetilde{M} e^{\delta t_m} \quad \textit{for all } m \in \mathbb{N},$$

where

$$\Psi_{11} = \alpha P - Q_1 C - C Q_1^T - U_1 \Sigma_1, \qquad \Psi_{12} = P - Q_1 - \beta C Q_1^T,$$

$$\Psi_{14} = Q_1 \widetilde{A} + U_1 \Sigma_2, \qquad \Psi_{22} = -\beta Q_1 - \beta Q_1^T + \gamma^2 \frac{e^{\alpha \widetilde{\tau}} - 1}{\alpha} T_{22},$$

$$\Psi_{33} = \widetilde{\tau} T_{11} - \gamma T_{12} - \gamma T_{12}^T - U_2 \Sigma_1, \qquad H = \mathrm{diag}(\widetilde{h}_1 \widehat{h}_1, \dots, \widetilde{h}_n \widehat{h}_n),$$

μ_k and $\mu_{\min}$ denote the largest eigenvalue of the matrix $(I - D_k) P (I - D_k)$ and the smallest eigenvalue of the matrix P, respectively.

Proof. Consider the Lyapunov–Krasovskiĭ functional $V = V_1 + V_2 + V_3 + V_4$, where, by putting $v(t) = V(t, e(t))$ and $v_i(t) = V_i(t, e(t))$ for $i \in \{1, 2, 3, 4\}$,

$$v_1(t) := e^{\alpha t} e^T(t) P e(t),$$

$$v_2(t) := \int_0^t e^{\alpha u} \int_{u - \tau(u)}^{u} \begin{pmatrix} e(u - \tau(u)) \\ \gamma \dot{e}(s) \end{pmatrix}^T T \begin{pmatrix} e(u - \tau(u)) \\ \gamma \dot{e}(s) \end{pmatrix} \mathrm{d}s \mathrm{d}u,$$

$$v_3(t) := \gamma^2 \int_{-\widetilde{\tau}}^{0} \int_{t+u}^{t} e^{\alpha(s-u)} \dot{e}^T(s) T_{22} \dot{e}(s) \mathrm{d}s \mathrm{d}u,$$

$$v_4(t) := \sum_{j=1}^{n} q_j^{(2)} \widetilde{h}_j \int_0^{\infty} h_j(u) \int_{t-u}^{t} e^{\alpha(s+u)} g_j^2(e_j(s)) \mathrm{d}s \mathrm{d}u.$$

Calculating the time derivatives of V_1, V_2, V_3, V_4 along the solutions of (2.4) in the interval $[t_{k-1}, t_k)$, $k \in \mathbb{N}$, we have

$$\begin{aligned} \dot{v}_1(t) &= \alpha e^{\alpha t} e^T(t) P e(t) + 2 e^{\alpha t} e^T(t) P \dot{e}(t) \\ &= \alpha e^{\alpha t} e^T(t) P e(t) + 2 e^{\alpha t} e^T(t) P \dot{e}(t) \\ &\quad + 2 e^{\alpha t} (e(t) + \beta \dot{e}(t))^T Q_1 \left\{ -\dot{e}(t) - C e(t) + \widetilde{A} g(e(t)) + \widetilde{B} g(e(t - \tau(t))) + W \int_{-\infty}^{t} h(t-s) g(e(s)) \mathrm{d}s \right\} \\ &= e^{\alpha t} \Big\{ e^T(t) \left[\alpha P - 2 Q_1 C\right] e(t) + 2 e^T(t) \left[P - Q_1 - \beta C Q_1^T\right] \dot{e}(t) \\ &\quad + 2 e^T(t) Q_1 \widetilde{A} g(e(t)) + 2 e^T(t) Q_1 \widetilde{B} g(e(t - \tau(t))) \\ &\quad + 2 e^T(t) Q_1 W \int_{-\infty}^{t} h(t-s) g(e(s)) \mathrm{d}s + \dot{e}^T(t) \left[-2\beta Q_1\right] \dot{e}(t) + 2 \dot{e}^T(t) \left[\beta Q_1 \widetilde{A}\right] g(e(t)) \\ &\quad + 2 \dot{e}^T(t) \left[\beta Q_1 \widetilde{B}\right] g(e(t - \tau(t))) + 2 \dot{e}^T(t) \left[\beta Q_1 W\right] \int_{-\infty}^{t} h(t-s) g(e(s)) \mathrm{d}s \Big\}, \end{aligned} \tag{3.2}$$

$$\begin{aligned} \dot{v}_2(t) &= e^{\alpha t} \int_{t - \tau(t)}^{t} \begin{pmatrix} e(t - \tau(t)) \\ \gamma \dot{e}(s) \end{pmatrix}^T \begin{pmatrix} T_{11} & T_{12} \\ \star & T_{22} \end{pmatrix} \begin{pmatrix} e(t - \tau(t)) \\ \gamma \dot{e}(s) \end{pmatrix} \mathrm{d}s \\ &= e^{\alpha t} \Big\{ \tau(t) e^T(t - \tau(t)) T_{11} e(t - \tau(t)) + 2\gamma e^T(t) T_{12}^T e(t - \tau(t)) \\ &\quad - 2\gamma e^T(t - \tau(t)) T_{12}^T e(t - \tau(t)) + \gamma^2 \int_{t - \tau(t)}^{t} \dot{e}^T(s) T_{22} \dot{e}(s) \mathrm{d}s \Big\} \\ &\le e^{\alpha t} \left\{ e^T(t - \tau(t)) \left[\widetilde{\tau} T_{11} - 2\gamma T_{12}^T\right] e(t - \tau(t)) + 2\gamma e^T(t) T_{12}^T e(t - \tau(t)) + \gamma^2 \int_{t - \widetilde{\tau}}^{t} \dot{e}^T(s) T_{22} \dot{e}(s) \mathrm{d}s \right\}, \end{aligned} \tag{3.3}$$

and

$$\begin{aligned}\dot{v}_3(t) &= \gamma^2 \int_{-\widetilde{\tau}}^{0} e^{\alpha(t-u)} \dot{e}^T(t) T_{22} \dot{e}(t) du - \gamma^2 \int_{-\widetilde{\tau}}^{0} e^{\alpha t} \dot{e}^T(t+u) T_{22} \dot{e}(t+u) du \\ &= \gamma^2 e^{\alpha t} \left\{ \frac{e^{\alpha \widetilde{\tau}} - 1}{\alpha} \dot{e}^T(t) T_{22} \dot{e}(t) - \int_{t-\widetilde{\tau}}^{t} \dot{e}^T(s) T_{22} \dot{e}(s) ds \right\}. \end{aligned} \tag{3.4}$$

Also, by the well-known Cauchy–Schwarz inequality, we obtain

$$\begin{aligned}\dot{v}_4(t) &= \sum_{j=1}^{n} q_j^{(2)} \widetilde{h}_j \int_0^\infty h_j(u) e^{\alpha(t+u)} g_j^2(e_j(t)) du - \sum_{j=1}^{n} q_j^{(2)} \widetilde{h}_j \int_0^\infty h_j(u) e^{\alpha t} g_j^2(e_j(t-u)) du \\ &\le e^{\alpha t} \left\{ g^T(e(t)) Q_2 H g(e(t)) - \sum_{j=1}^{n} q_j^{(2)} \int_0^\infty h_j(u) du \int_0^\infty h_j(u) g_j^2(e_j(t-u)) du \right\} \\ &\le e^{\alpha t} \left\{ g^T(e(t)) Q_2 H g(e(t)) - \sum_{j=1}^{n} q_j^{(2)} \left(\int_0^\infty h_j(u) g_j(e_j(t-u)) du \right)^2 \right\} \\ &= e^{\alpha t} \left\{ g^T(e(t)) Q_2 H g(e(t)) - \left(\int_{-\infty}^{t} h(t-s) g(e(s)) ds \right)^T Q_2 \left(\int_{-\infty}^{t} h(t-s) g(e(s)) ds \right) \right\}. \end{aligned} \tag{3.5}$$

Using now

$$e^{\alpha t} \left\{ \begin{pmatrix} e(t) \\ g(e(t)) \end{pmatrix}^T \begin{pmatrix} -U_1 \Sigma_1 & U_1 \Sigma_2 \\ \star & -U_1 \end{pmatrix} \begin{pmatrix} e(t) \\ g(e(t)) \end{pmatrix} + \begin{pmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{pmatrix}^T \begin{pmatrix} -U_2 \Sigma_1 & U_2 \Sigma_2 \\ \star & -U_2 \end{pmatrix} \begin{pmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{pmatrix} \right\} \ge 0$$

(which holds by [32,40] since U_1 and U_2 are positive definite diagonal matrices) in (3.2)–(3.5), we get

$$\begin{aligned} e^{-\alpha t} \dot{v}(t) \le\ & e^T(t) \left[\alpha P - Q_1 C - C Q_1^T \right] e(t) + 2 e^T(t) \left[P - Q_1 - \beta C Q_1^T \right] \dot{e}(t) \\ & + 2 e^T(t) Q_1 \widetilde{A} g(e(t)) + 2 e^T(t) Q_1 \widetilde{B} g(e(t-\tau(t))) \\ & + 2 e^T(t) Q_1 W \int_{-\infty}^{t} h(t-s) g(e(s)) ds + \dot{e}^T(t) \left[-\beta Q_1 - \beta Q_1^T \right] \dot{e}(t) \\ & + 2 \dot{e}^T(t) \left[\beta Q_1 \widetilde{A} \right] g(e(t)) + 2 \dot{e}^T(t) \left[\beta Q_1 \widetilde{B} \right] g(e(t-\tau(t))) + 2 \dot{e}^T(t) \left[\beta Q_1 W \right] \int_{-\infty}^{t} h(t-s) g(e(s)) ds \\ & + e^T(t-\tau(t)) \left[\widetilde{\tau} T_{11} - \gamma T_{12}^T - \gamma T_{12} \right] e(t-\tau(t)) + 2\gamma e^T(t) T_{12}^T e(t-\tau(t)) + \gamma^2 \frac{e^{\alpha \widetilde{\tau}} - 1}{\alpha} \dot{e}^T(t) T_{22} \dot{e}(t) \\ & + g^T(e(t)) Q_2 H g(e(t)) - \left(\int_{-\infty}^{t} h(t-s) g(e(s)) ds \right)^T Q_2 \left(\int_{-\infty}^{t} h(t-s) g(e(s)) ds \right) \\ & + \begin{pmatrix} e(t) \\ g(e(t)) \end{pmatrix}^T \begin{pmatrix} -U_1 \Sigma_1 & U_1 \Sigma_2 \\ \star & -U_1 \end{pmatrix} \begin{pmatrix} e(t) \\ g(e(t)) \end{pmatrix} \\ & + \begin{pmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{pmatrix}^T \begin{pmatrix} -U_2 \Sigma_1 & U_2 \Sigma_2 \\ \star & -U_2 \end{pmatrix} \begin{pmatrix} e(t-\tau(t)) \\ g(e(t-\tau(t))) \end{pmatrix} \\ =\ & \xi^T(t) \Psi \xi(t), \end{aligned}$$

where Ψ is as in (3.1) and

$$\xi(t) = \left(e(t),\ \dot{e}(t),\ e(t-\tau(t)),\ g(e(t)),\ g(e(t-\tau(t))),\ \int_{-\infty}^{t} h(t-s) g(e(s)) ds \right)^T.$$

From (3.1), we know that the matrix Ψ is negative definite, which implies

$$\dot{v}(t) \le 0 \quad \text{for all } t \in [t_{k-1}, t_k),\ k \in \mathbb{N}. \tag{3.6}$$

In addition, we note that for any $k \in \mathbb{N}$

$$\begin{aligned} v_1(t_k) &= e^{\alpha t_k} e^T(t_k) P e(t_k) = e^{\alpha t_k} e^T(t_k^-)(I - D_k) P (I - D_k) e(t_k^-) \\ &\le e^{\alpha t_k} \frac{\mu_k}{\mu_{\min}} e^T(t_k^-) P e(t_k^-) = \frac{\mu_k}{\mu_{\min}} v_1(t_k^-) \end{aligned}$$

and

$$v_2(t_k) = v_2(t_k^-), \qquad v_3(t_k) = v_3(t_k^-), \qquad v_4(t_k) = v_4(t_k^-),$$

which yields

$$v(t_k) \le \max\left\{\frac{\mu_k}{\mu_{\min}}, 1\right\} v(t_k^-) \quad \text{for } k \in \mathbb{N}. \tag{3.7}$$

By mathematical induction, from (3.6) and (3.7), we can obtain for $t \in [t_m, t_{m+1}), m \in \mathbb{N}$

$$\mathrm{e}^{\alpha t}\mu_{\min}\,\|e(t)\|^2 \le v(t) \le v(0)\prod_{k=1}^{m}\max\left\{\frac{\mu_k}{\mu_{\min}}, 1\right\},$$

which implies that

$$\|e(t)\|^2 \le \frac{\widetilde{M}}{\mu_{\min}} v(0)\mathrm{e}^{-\alpha t}\mathrm{e}^{\delta t_m} \quad \text{for all } t \in [t_m, t_{m+1}), m \in \mathbb{N}.$$

Hence

$$\|e(t)\|^2 \le \frac{\widetilde{M}}{\mu_{\min}} v(0)\mathrm{e}^{-(\alpha-\delta)t} \quad \text{for all } t > 0. \tag{3.8}$$

On the other hand, we compute that

$$\begin{aligned}
v(0) &= e^T(0)Pe(0) + \gamma^2\int_{-\tilde{\tau}}^{0}\int_{u}^{0}\mathrm{e}^{\alpha(s-u)}\dot{e}^T(s)T_{22}\dot{e}(s)\mathrm{d}s\mathrm{d}u + \sum_{j=1}^{n} q_j^{(2)}\widetilde{h}_j\int_0^{\infty} h_j(u)\int_{-u}^{0}\mathrm{e}^{\alpha(s+u)}g_j^2(e_j(s))\mathrm{d}s\mathrm{d}u\\
&\le \mu_{\max}\,\|\varphi-\phi\|_{\tilde{\tau}}^2 + \lambda_{\max}\gamma^2\frac{\mathrm{e}^{\alpha\tilde{\tau}}-1-\alpha\tilde{\tau}}{\alpha^2}\,\|\varphi-\phi\|_{\tilde{\tau}}^2 + \frac{1}{\alpha}\sum_{j=1}^{n} q_j^{(2)}\widetilde{h}_j\sigma_j^2(\widehat{h}_j-\widetilde{h}_j)\,\|\varphi-\phi\|_{\tilde{\tau}}^2\\
&\le \left\{\mu_{\max} + \lambda_{\max}\gamma^2\frac{\mathrm{e}^{\alpha\tilde{\tau}}-1-\alpha\tilde{\tau}}{\alpha^2} + \frac{1}{\alpha}\sum_{j=1}^{n} q_j^{(2)}\widetilde{h}_j\sigma_j^2(\widehat{h}_j-\widetilde{h}_j)\right\}\|\varphi-\phi\|_{\tilde{\tau}}^2,
\end{aligned}$$

where $\sigma_j = \max\{|\sigma_j^-|,\ |\sigma_j^+|\}, j \in \Lambda$, $\mu_{\max}$ and $\lambda_{\max}$ denote the maximum eigenvalues of the matrix P and T_{22}, respectively, which when used in (3.8) provides

$$\|e(t)\| \le M\,\|\varphi-\phi\|_{\tilde{\tau}}\,\mathrm{e}^{-\frac{\alpha-\delta}{2}t} \quad \text{for all } t > 0,$$

where

$$M = \sqrt{\frac{\widetilde{M}}{\mu_{\min}}\left\{\mu_{\max} + \lambda_{\max}\gamma^2\left(\frac{\mathrm{e}^{\alpha\tilde{\tau}}-1-\alpha\tilde{\tau}}{\alpha^2}\right) + \frac{1}{\alpha}\sum_{j=1}^{n} q_j^{(2)}\widetilde{h}_j\sigma_j^2(\widehat{h}_j-\widetilde{h}_j)\right\}} \ge 1.$$

Using Definition 2.3, it can be concluded that system (2.4) is exponentially stable, i.e., the models (2.1) and (2.2) achieve exponential synchronization. This completes the proof. □

Remark 3.2. In particular, it should be pointed out that the scalars β and γ in Theorem 3.1 play an important rôle to ensure the reliability and rationality of the LMIs.

In order to estimate the gain matrices K_1 and K_2, we give the following useful results.

Corollary 3.3. *Assume* (H_1)–(H_3). *Then the models* (2.1) *and* (2.2) *are exponentially synchronized if there exist five constants* $\alpha \in (0,\eta)$, $\beta, \gamma > 0$, $\delta \in [0,\alpha)$, $\widetilde{M} > 1$, *three* $n\times n$*-matrices* $P > 0$, Y_1, Y_2, *an invertible* $n\times n$*-matrix* Q_1, *three diagonal* $n\times n$*-matrices* $Q_2 > 0$, $U_1 > 0$, $U_2 > 0$, *and a* $2n\times 2n$*-matrix* $\begin{pmatrix} T_{11} & T_{12}\\ \star & T_{22}\end{pmatrix} > 0$ *such that*

$$\begin{pmatrix}
\Psi_{11} & \Psi_{12} & \gamma T_{12}^T & \Psi_{14} & Q_1B+Y_2 & Q_1W\\
\star & \Psi_{22} & 0 & \beta Q_1A+\beta Y_1 & \beta Q_1B+\beta Y_2 & \beta Q_1W\\
\star & \star & \Psi_{33} & 0 & U_2\Sigma_2 & 0\\
\star & \star & \star & Q_2H-U_1 & 0 & 0\\
\star & \star & \star & \star & -U_2 & 0\\
\star & \star & \star & \star & \star & -Q_2
\end{pmatrix} < 0$$

and

$$\prod_{k=1}^{m} \max\left\{\frac{\mu_k}{\mu_{\min}}, 1\right\} \leq \widetilde{M} e^{\delta t_m} \quad \textit{for all } m \in \mathbb{N},$$

where

$$\Psi_{11} = \alpha P - Q_1 C - C Q_1^T - U_1 \Sigma_1, \qquad \Psi_{12} = P - Q_1 - \beta C Q_1^T,$$
$$\Psi_{14} = Q_1 A + Y_1 + U_1 \Sigma_2, \qquad \Psi_{22} = -\beta Q_1 - \beta Q_1^T + \gamma^2 \frac{e^{\alpha\widetilde{\tau}} - 1}{\alpha} T_{22},$$
$$\Psi_{33} = \widetilde{\tau} T_{11} - \gamma T_{12} - \gamma T_{12}^T - U_2 \Sigma_1, \qquad H = \mathrm{diag}(\widetilde{h}_1 \widehat{h}_1, \ldots, \widetilde{h}_n \widehat{h}_n),$$

μ_k and $\mu_{\min}$ denote the largest eigenvalue of the matrix $(I - D_k) P (I - D_k)$ and the smallest eigenvalue of the matrix P, respectively.

Proof. Let $K_i = Q_1^{-1} Y_i$ in Theorem 3.1. Then the result follows from Theorem 3.1. □

When $D_k = 0$, models (2.1) and (2.2) become neural networks without impulsive effects. By Corollary 3.3, we have the following result.

Corollary 3.4. *Assume* (H_1)–(H_3). *Then models* (2.1) *and* (2.2) *without impulsive effects are exponentially synchronized if there exist three constants $\alpha \in (0, \eta)$, $\beta, \gamma > 0$, three $n \times n$-matrices $P > 0$, Y_1, Y_2, an invertible $n \times n$-matrix Q_1, three diagonal $n \times n$-matrices $Q_2 > 0$, $U_1 > 0$, $U_2 > 0$, and a $2n \times 2n$-matrix $\begin{pmatrix} T_{11} & T_{12} \\ \star & T_{22} \end{pmatrix} > 0$ such that*

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} & \gamma T_{12}^T & \Psi_{14} & Q_1 B + Y_2 & Q_1 W \\ \star & \Psi_{22} & 0 & \beta Q_1 A + \beta Y_1 & \beta Q_1 B + \beta Y_2 & \beta Q_1 W \\ \star & \star & \Psi_{33} & 0 & U_2 \Sigma_2 & 0 \\ \star & \star & \star & Q_2 H - U_1 & 0 & 0 \\ \star & \star & \star & \star & -U_2 & 0 \\ \star & \star & \star & \star & \star & -Q_2 \end{pmatrix} < 0,$$

where

$$\Psi_{11} = \alpha P - Q_1 C - C Q_1^T - U_1 \Sigma_1, \qquad \Psi_{12} = P - Q_1 - \beta C Q_1^T,$$
$$\Psi_{14} = Q_1 A + Y_1 + U_1 \Sigma_2, \qquad \Psi_{22} = -\beta Q_1 - \beta Q_1^T + \gamma^2 \frac{e^{\alpha\widetilde{\tau}} - 1}{\alpha} T_{22},$$
$$\Psi_{33} = \widetilde{\tau} T_{11} - \gamma T_{12} - \gamma T_{12}^T - U_2 \Sigma_1, \qquad H = \mathrm{diag}(\widetilde{h}_1 \widehat{h}_1, \ldots, \widetilde{h}_n \widehat{h}_n).$$

Remark 3.5. When $W = 0$, models (2.1) and (2.2) become the neural network [37]

$$\begin{cases} \dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t - \tau(t))) + I(t), & t \neq t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = -D_k x(t_k^-), & k \in \mathbb{N}, \\ x(s) = \phi(s), & s \in (-\infty, 0] \end{cases} \tag{3.9}$$

and

$$\begin{cases} \dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \tau(t))) + I(t) + u(t), & t \neq t_k, \\ \Delta y(t_k) = y(t_k) - y(t_k^-) = -D_k y(t_k^-), & k \in \mathbb{N}, \\ y(s) = \varphi(s), & s \in (-\infty, 0]. \end{cases} \tag{3.10}$$

The out-coupling controller (2.3) is still considered.

Using Theorem 3.1, it is easy to obtain the following corollary.

Corollary 3.6. *Assume* (H_1) *and* (H_3). *Then the models* (3.9) *and* (3.10) *are exponentially synchronized if there exist five constants $\alpha \in (0, \eta)$, $\beta, \gamma > 0$, $\delta \in [0, \alpha)$, $\widetilde{M} > 1$, an $n \times n$-matrix $P > 0$, an $n \times n$-matrix Q_1, two diagonal $n \times n$-matrices $U_1 > 0$, $U_2 > 0$, and a $2n \times 2n$-matrix $\begin{pmatrix} T_{11} & T_{12} \\ \star & T_{22} \end{pmatrix} > 0$ such that*

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} & \gamma T_{12}^T & \Psi_{14} & Q_1 \widetilde{B} \\ \star & \Psi_{22} & 0 & \beta Q_1 \widetilde{A} & \beta Q_1 \widetilde{B} \\ \star & \star & \Psi_{33} & 0 & U_2 \Sigma_2 \\ \star & \star & \star & -U_1 & 0 \\ \star & \star & \star & \star & -U_2 \end{pmatrix} < 0$$

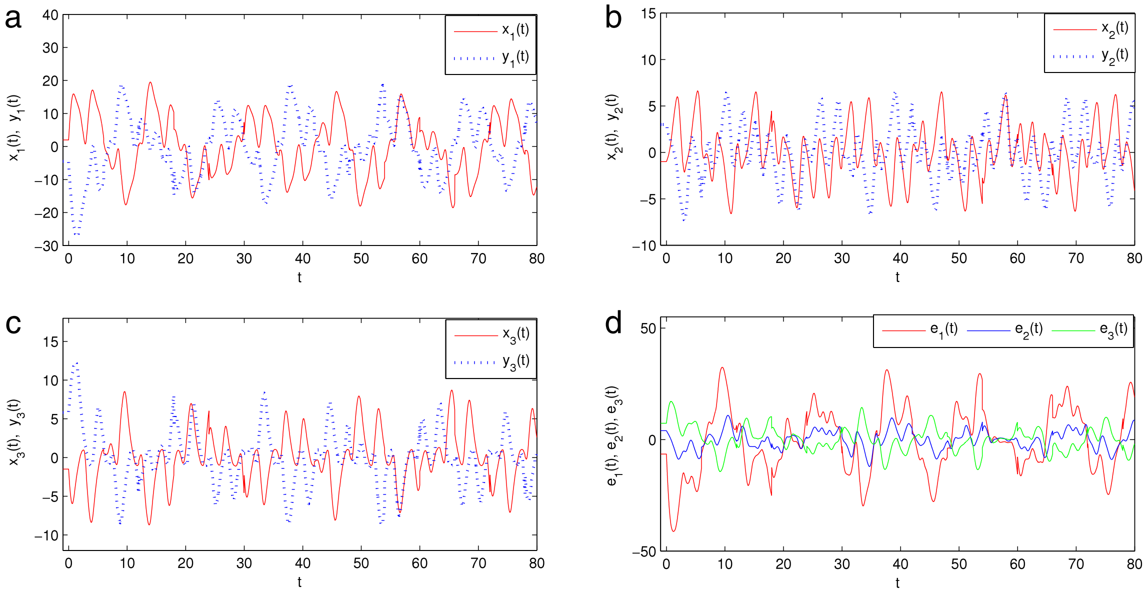

Fig. 4.1. State trajectories and error trajectories of drive system (4.1) and response system (4.2) without control input.

and

$$\prod_{k=1}^{m} \max\left\{\frac{\mu_k}{\mu_{\min}},\ 1\right\} \le \widetilde{M} e^{\delta t_m} \quad \textit{for all } m \in \mathbb{N},$$

where

$$\Psi_{11} = \alpha P - Q_1 C - C Q_1^T - U_1 \Sigma_1, \qquad \Psi_{12} = P - Q_1 - \beta C Q_1^T,$$

$$\Psi_{14} = Q_1 \widetilde{A} + U_1 \Sigma_2, \qquad \Psi_{22} = -\beta Q_1 - \beta Q_1^T + \gamma^2 \frac{e^{\alpha\widetilde{\tau}} - 1}{\alpha} T_{22},$$

$$\Psi_{33} = \widetilde{\tau} T_{11} - \gamma T_{12} - \gamma T_{12}^T - U_2 \Sigma_1,$$

and μ_k *and* $\mu_{\min}$ *denote the largest eigenvalue of the matrix* $(I - D_k)P(I - D_k)$ *and the smallest eigenvalue of the matrix* P*, respectively.*

Remark 3.7. In [37], Gao et al. have considered the exponential synchronization problem of models (3.9) and (3.10) via state coupling $u(t) = Ke(t)$. Although the coupling form in their paper has an advantage of easy implementation, the impact from the network delays is ignored. Moreover, the conditions on time-varying delays are too restrictive (see Remark 2.1) and the impulsive disturbance was also not addressed. Hence, our results complement and improve those in [37].

Remark 3.8. Here we would like to point out that it is possible to design exponential synchronization LMI-based conditions for the model (2.1) and (2.2) via control input vector with state and time-delay state feedback by the ideas in this paper. The results will appear in the near future.

4. Illustrative example

In this section, an example is provided in order to illustrate the effectiveness of the obtained results.

Example 4.1. Consider the chaotic neural network model with impulses

$$\begin{cases} \dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + W\displaystyle\int_{-\infty}^{t} h(t-s)f(x(s))\mathrm{d}s + I(t), \quad t \neq t_k, \\ \Delta x(t_k) = x(t_k) - x(t_k^-) = -D_k x(t_k^-), \quad k \in \mathbb{N}, \\ x(s) = \phi(s), \quad s \in (-\infty, 0], \end{cases} \tag{4.1}$$

where the data are given as

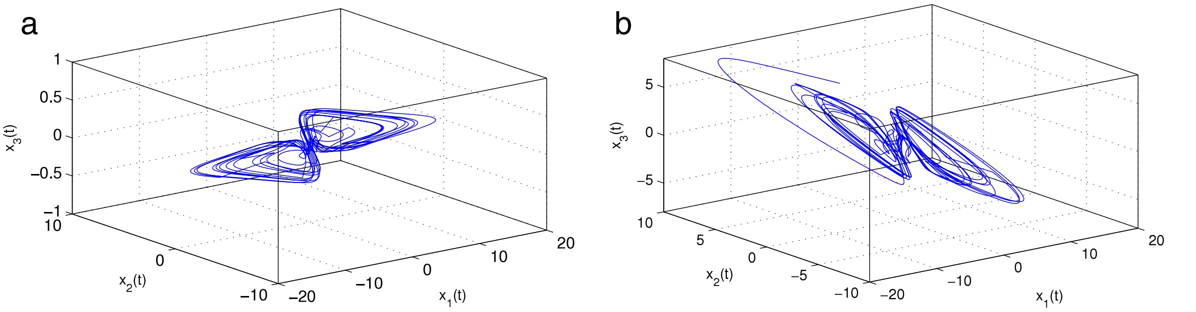

Fig. 4.2. (a) Chaotic behavior of drive system (4.1) in phase space with initial condition $\phi(s) = (2, -1, -1.5)^T$, $s \in (-\infty, 0]$. (b) Chaotic behavior of response system (4.2) in phase space without control input with initial condition $\varphi(s) = (-4.5, 3, 5.8)^T$, $s \in (-\infty, 0]$.

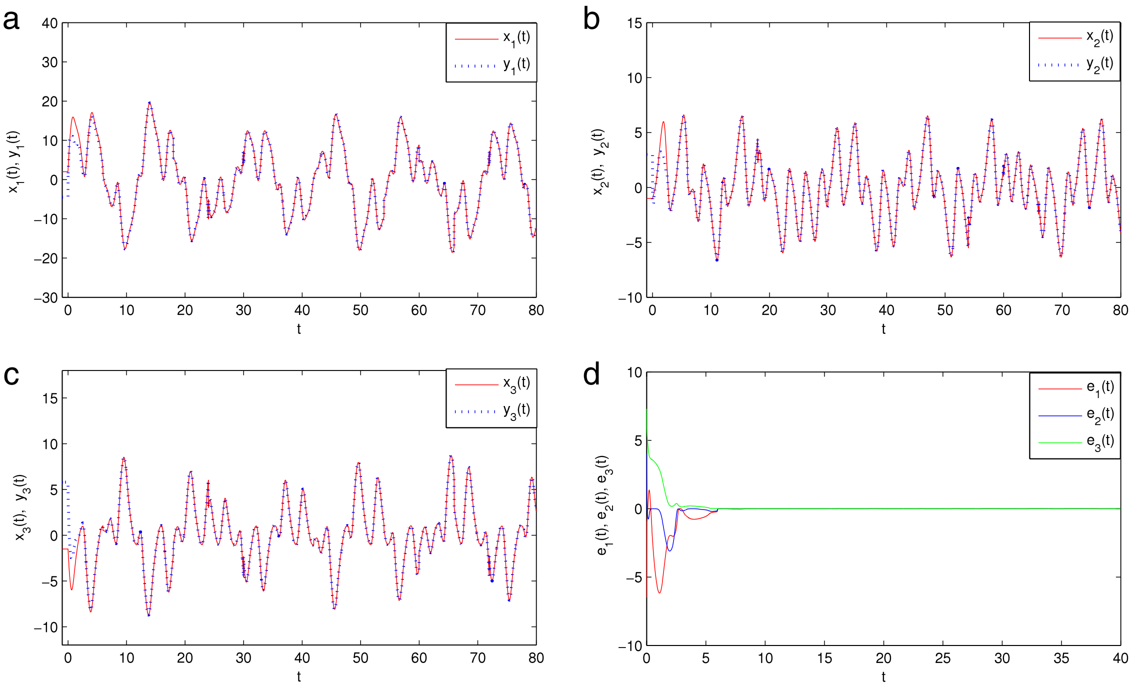

Fig. 4.3. State trajectories and error trajectories of drive system (4.1) and response system (4.2) with control input (4.3).

$$\phi(s) = (2, -1, -1.5)^T, \quad s \in (-\infty, 0],$$
$$f(x) = \tanh(x), \qquad h(s) = 0.35e^{-s}, \qquad I(t) = 0, \qquad \tau(t) = 0.24, \qquad t_k = 6k, \quad k \in \mathbb{N},$$
$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad A = \begin{pmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 6.3 & -8.5 & -3 \\ -3 & 1.2 & -5.5 \\ -3.2 & 4.5 & -2.3 \end{pmatrix},$$
$$W = \begin{pmatrix} 2 & -13.3 & -20.1 \\ -3.3 & 10.5 & -10.37 \\ -3.15 & 3.23 & 0.95 \end{pmatrix}, \qquad D_k = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}.$$

The corresponding response system is designed as

$$\begin{cases} \dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t-\tau(t))) + W\displaystyle\int_{-\infty}^{t} h(t-s)f(y(s))\mathrm{d}s + I(t) + u(t), & t \neq t_k, \\ \Delta y(t_k) = y(t_k) - y(t_k^-) = -D_k y(t_k^-), & k \in \mathbb{N}, \\ y(s) = \varphi(s), & s \in (-\infty, 0], \end{cases} \tag{4.2}$$

where u is given by (2.3) and the initial condition is

$$\varphi(s) = (-4.5, 3, 5.8)^T, \quad s \in (-\infty, 0].$$

Let $\eta = 0.65$, $\alpha = 0.64$, $\beta = 0.01$, $\gamma = 5$. Using the Matlab LMI toolbox, we can obtain the following feasible solutions to the LMIs in Corollary 3.3:

$$P = \begin{pmatrix} 24.6873 & -22.5952 & 109.2845 \\ -22.5952 & 49.7123 & -150.0789 \\ 109.2845 & -150.0789 & 800.6915 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} 24.6888 & -22.6268 & 109.4215 \\ -22.6268 & 49.7683 & -150.2631 \\ 109.4215 & -150.2631 & 801.6713 \end{pmatrix}, \qquad Q_2 = 10^3 \times \begin{pmatrix} 4.3265 & 0 & 0 \\ 0 & 4.3265 & 0 \\ 0 & 0 & 4.3265 \end{pmatrix},$$

$$Y_1 = 10^3 \times \begin{pmatrix} -1.1675 & -0.3772 & -0.1301 \\ -0.2929 & -0.8828 & 0.2966 \\ 1.9476 & -3.0121 & -2.5288 \end{pmatrix}, \qquad Y_2 = 10^3 \times \begin{pmatrix} 0.1260 & -0.2554 & 0.2013 \\ -0.1890 & 0.4233 & -0.1398 \\ 1.4252 & -2.4971 & 1.3449 \end{pmatrix},$$

$$U_1 = 10^3 \times \begin{pmatrix} 2.8369 & 0 & 0 \\ 0 & 2.8369 & 0 \\ 0 & 0 & 2.8369 \end{pmatrix}, \qquad U_2 = \begin{pmatrix} 1.3948 & 0 & 0 \\ 0 & 1.3948 & 0 \\ 0 & 0 & 1.3948 \end{pmatrix},$$

$$T_{11} = \begin{pmatrix} 11.9500 & -15.0231 & 70.6648 \\ -15.0231 & 31.0565 & -96.5328 \\ 70.6648 & -96.5328 & 514.3971 \end{pmatrix}, \qquad T_{12} = \begin{pmatrix} 0.5214 & -0.6467 & 3.0664 \\ -0.6467 & 1.3137 & -4.1953 \\ 3.0664 & -4.1953 & 22.3159 \end{pmatrix},$$

$$T_{22} = \begin{pmatrix} 0.0403 & -0.0526 & 0.2539 \\ -0.0526 & 0.0992 & -0.3486 \\ 0.2539 & -0.3486 & 1.8436 \end{pmatrix}.$$

Consequently, the controller gain matrices K_1 and K_2 are designed as

$$K_1 = Q_1^{-1}Y_1 = \begin{pmatrix} -149.4058 & -11.3036 & 20.7033 \\ -11.3036 & -68.1059 & -6.1800 \\ 20.7033 & -14.9800 & -7.1386 \end{pmatrix}, \qquad K_2 = Q_1^{-1}Y_2 = \begin{pmatrix} -6.3771 & 8.4931 & 3.0092 \\ 2.9931 & -1.2397 & 5.4935 \\ 3.2092 & -4.5065 & 2.2965 \end{pmatrix}. \tag{4.3}$$

Moreover, let $\widetilde{M} = 1$ and $\delta = 0.54 < \alpha$. Then

$$\prod_{k=1}^{m} \frac{\mu_k}{\mu_{\min}} \approx 22.7007^m < 25.5337^m = e^{\delta t_m} \quad \text{for all } m \in \mathbb{N}.$$

By Corollary 3.3, models (4.1) and (4.2) are exponentially synchronized. The simulation results are illustrated in Fig. 4.3 (a)–(d) in which the controller designed in (4.3) is applied.

Remark 4.2. In the simulations, we choose the time step size $h = 0.01$ and time segment $T = 80$. In addition, we should point out that for simplicity of our computer simulations, the delay kernel h is used as $h(s) = 0.35e^{-s}$ for $s \in [0, 20]$ and $h(s) = 0$ for $s > 20$. The simulation results can be described as follows. Fig. 4.1(a)–(d) shows the state trajectories and the error trajectories between the drive system (4.1) and the response system (4.2) without control input. One may observe that the drive system (4.1) and the response system (4.2) without control input cannot be synchronized, not to mention exponential synchronization. Fig. 4.2(a)–(b) depicts the chaotic behavior in phase space of the drive system (4.1) and the response system (4.2) without control input, respectively. Fig. 4.3(a)–(d) shows the state trajectories and error trajectories of the drive system (4.1) and the response system (4.2) with control input (4.3). From the simulations, we can find that exponential synchronization of system (4.1) is realized via the feedback gain matrices K_1 and K_2, and those simulations match the obtained results perfectly.

5. Conclusions

In this paper, we have investigated exponential synchronization of chaotic neural networks with mixed delays and impulsive effects via output coupling with delay feedback. By using a Lyapunov–Krasovskiĭ functional, a drive–response concept, and a linear matrix inequality (LMI) approach, we have proposed several LMI-based criteria to guarantee exponential synchronization of the neural network. Also, the estimation gains can be easily obtained by solving the LMIs. It is convenient to apply these criteria to real networks. Moreover, the obtained results complement and improve some existing results. An example and its numerical simulation are also given in order to demonstrate the effectiveness of the theoretical results.

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