Chapter 4

Continuous Random Variables

cdf

The cumulative distribution function (cdf) of a continuous rv X is defined for every number x by

$F(x)=P(X\leq x)$

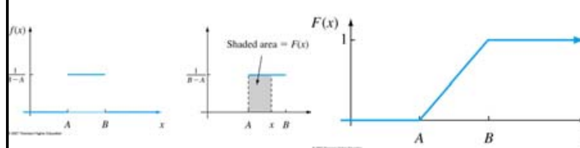
Example

Define a function f by $f(x)=0.5+x$ if $0\leq x\leq 1$ and $f(x)=0$ otherwise. Suppose that f is the pdf of a continuous rv X.

- **Find the cdf of X.**
- **What is the probability that X is less than 0.5?**
- **What is the probability that X is bigger than 0.5?**
- **What is the probability that X is between 3/4 and 5/6?**

Example

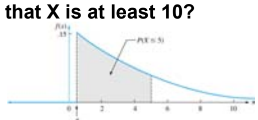
Find the cdf of a uniform continuous rv.



Example

Suppose the pdf of a continuous rv X is given by $f(x)=0.15\exp(-0.15(x-0.5))$ if $x\geq 0.5$ and $f(x)=0$ otherwise.

- **Find the cdf of X.**
- **What is the probability the X is at most 5?**
- **What is the probability that X is at least 10?**



cdf - pdf

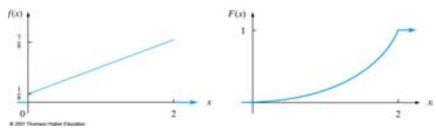
The pdf f of a continuous rv X can be obtained from the pdf F when F' exists:

$f(x)=F'(x)$

Example

Suppose the cdf of a continuous rv X is given by F(x)=0 for x<0, $F(x)=x/8+3x^2/16$ for $0 \le x \le 2$, and F(x)=1 for x>2.

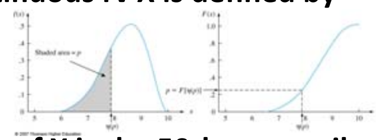
Find the pdf of X.



Percentile and Median

Let $0 \le p \le 1$. The (100p)th percentile η(p) of a continuous rv X is defined by

F(η(p))=p.



The median of X is the 50th percentile.

Example

Suppose the cdf of a continuous rv X is given by F(x)=0 for x<0, $F(x)=x/8+3x^2/16$ for $0 \le x \le 2$, and F(x)=1 for x>2.

- **Find the 20th percentile of X.**
- **Find the 70th percentile of X.**
- **Find the median of X.**

Expected Value

Let X be a continuous rv with pdf f. The expected value or mean value of X is defined by

$$E(X) = \mu_X = \mu = \int_{-\infty}^{\infty} x \cdot f(x)dx$$

Example

Find the expected value of the continuous uniform random variable.

Example

Suppose the pdf of a continuous rv X is given by f(x)=0.15exp(-0.15(x-0.5)) if $x \ge 0.5$ and f(x)=0 otherwise.

Find E(X).

Expected Value of a Function

Let X be a continuous rv with pdf f. The **expected value** of a function h(X) is

$$E(h(X)) = \mu_{h(X)} = \int_{-\infty}^{\infty} h(x) \cdot f(x)dx$$

Properties of Expectation

1. E(aX)=aE(X) for all numbers a
2. E(X+b)=E(x)+b for all numbers b
3. E(aX+b)=aE(X)+b for all numbers a, b

Variance

Let X be a continuous rv with pdf f. The **variance** of X is defined by

$$V(X) = E((X - E(X))^2) = \sigma_X^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)dx$$

The root of the variance is called the **standard deviation (SD)** of X

Example

Find the variance of the continuous uniform random variable on [0,1].

A Shortcut Formula for the Variance

An alternative expression for the variance is

$$V(X)=E(X^2)-(E(X))^2$$

Example

Define a function f by f(x)=0.5+x if $0 \le x \le 1$ and f(x)=0 otherwise. Suppose that f is the pdf of a continuous rv X.

- **Find E(X).**
- **Find $E(X^2)$.**
- **Find V(X).**

Properties of the Variance

1. $V(aX+b)=a^2V(X)$ for all numbers a, b
2. $V(X+b)=V(x)$ for all numbers b
3. $V(aX)=a^2V(X)$ for all numbers a
4. $\sigma_{aX+b}=|a|\sigma_X$ for all numbers a, b
5. $\sigma_{aX}=|a|\sigma_X$ for all numbers a
6. $\sigma_{X+b}=\sigma_X$ for all numbers b