Chapter 2

Probability

2.1 Sample Spaces and Events

2.2 Axioms, Interpretations, and Properties of Probability

2.3 Counting Techniques

2.4 Conditional Probability

2.5 Independence

Axioms of Probability

1. The probability of any event is nonnegative.
2. The probability of the sample space is one.
3. If \(A_1, A_2, A_3, \ldots\) is an infinite collection of disjoint events, then

\[
P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)
\]

Properties of Probability

1. \(P(\emptyset) = 0\)
2. \(P(A) = 1 - P(A')\) for any event \(A\)
3. \(P(A) \leq 1\) for any event \(A\)
4. \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\) for any events \(A\) and \(B\)
5. \(P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)\)

for any events \(A, B\) and \(C\)

Example

A die is changed in such a manner that the probability to toss a number is proportional to that number.

a) Calculate the probabilities of the simple events.

b) Calculate the probabilities of the following events: An even number is tossed (event \(A\)). A prime number is tossed (event \(B\)). An odd number is tossed (event \(C\)).

c) Calculate \(P(A \cap B)\), \(P(B \cup C)\), \(P(B \cap C)\), \(P(A \cap B)\), \(P(A')\) and \(P(B')\).

Example

A job announcement asks applicants to indicate which of the three languages English, French and Russian they can speak. Altogether 190 people apply. 100 can speak English, 70 French, 67 Russian, 22 English and Russian, 20 English and French, 17 French and Russian. How many applicants can speak all three languages?