Chapter 2

Probability

2.1 Sample Spaces and Events
2.2 Axioms, Interpretations, and Properties of Probability
2.3 Counting Techniques
2.4 Conditional Probability
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Conditional Probability

For any two events A and B with P(B) > 0, the conditional probability of A given that B has occurred is defined by

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]

Example

400 women in a company are engineers, 100 women are not engineers, 200 men are engineers, and 800 men are not engineers. An employee is randomly chosen each month to win a price. What is the probability that it is an engineer? If it is known that the person is a woman, what is then the probability that it is an engineer?

Example

Among ten bulbs there are three that do not work. We select two bulbs successively (without replacing) and test them. By F₁ and F₂ we denote the events that the first and the second bulb, respectively, does not work. Find P(F₁), P(F₂), P(F₂|F₁), P(F₂|F₁'), P(F₂'|F₁), P(F₂'|F₁'), P(F₂ ∩ F₁), P(F₂' ∩ F₁'), P(F₂), and P(F₂').

Example

Assume an urn contains 4 black and 6 white balls. Two balls are selected randomly (without replacement). Let B₁, W₁, B₂, W₂ be the outcomes that the selected first ball is black, white, and the second ball is black, white, respectively. Find P(B₁|W₂) and P(B₁|B₂).
**Law of Total Probability**

Let $A_1, A_2, \ldots, A_k$ be mutually exclusive and exhaustive events. Then, for any event $B$, we have

$$P(B) = \sum_{i=1}^{k} P(B \mid A_i)P(A_i)$$

**Bayes’ Theorem**

Let $A_1, A_2, \ldots, A_k$ be mutually exclusive and exhaustive events. Then, for any event $B$, we have

$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i)P(A_i)}$$

**Example**

Among a group of people, 25% are vaccinated against flu. The probability that a person who is vaccinated gets the flu is 0.2. The probability that a person who is not vaccinated gets the flu is 0.3.

a) Suppose a person gets the flu. What is the probability that this person is vaccinated?

b) Suppose a person does not get the flu. What is the probability that this person is not vaccinated?