Chapter 3

Discrete Random Variables

3.1 Random Variables
3.2 Probability Distributions
3.3 Expected Values
3.4 The Binomial Probability Distribution
3.5 Hypergeometric and Negative Binomial Distributions
3.6 The Poisson Probability Distribution

Binomial Experiment

1. Each trial can result in one of the same two possible outcomes, success or failure
2. The trials are independent
3. The probability $p$ of success is constant from trial to trial
4. The experiment consists of a sequence of $n$ smaller experiment, called trials

Example

A coin that has been changed to have the probability of head equal to 1/3 is tossed (independently) three times. Let $X$ be the number of heads in the three trials.

Find the pmf of $X$.

Binomial RV

The binomial random variable $X$ associated with a binomial experiment is defined as the number of successes in the experiment. Its pmf is given by

$$p(x) = b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

Example

Each of seven randomly selected cola drinkers is given a glass containing cola S and one containing cola F. The glasses are identical in the appearance. Suppose the probability that a randomly selected individual prefers cola S to cola F is 0.6. Find the probability that

- Exactly four cola drinkers prefer cola S
- At least four cola drinkers prefer cola S
- At most one cola drinker prefers cola S.

Expectation and Variance of the Binomial RV

If $X$ is a binomial rv with parameters $n$ and $p$, then

$$E(X) = np$$

$$V(X) = np(1-p)$$
Example

Suppose that 75% of all purchases at a certain store are made with a credit card and let $X$ be the number among ten randomly selected purchases made with a credit card.

- Find $E(X)$
- Find $V(X)$