Chapter 4
Continuous Random Variables

4.1 Probability Density Functions
4.2 Cumulative Distribution Functions and Expected Values
4.3 The Normal Distribution
4.4 The Exponential and Gamma Distributions
4.5 Other Continuous Distributions

Normal RV

The normal (or Gaussian) rv $X$ with parameters $\mu$ and $\sigma > 0$ is the rv with the pdf

$$f(x; \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Expectation and Variance of the normal RV

If $X$ is a normal rv with parameters $\mu$ and $\sigma$, then

$$E(X) = \mu$$

$$V(X) = \sigma^2$$

Standard Normal RV

The standard normal (or Gaussian) rv $Z$ is the normal rv with $\mu = 0$ and $\sigma = 1$ and the pdf

$$f(z; 0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$
Example

(a) Find $P(Z \leq 1.25)$

(b) Find $P(Z > 1.25)$

(c) Find $P(Z \leq -1.25)$

Example

(d) Find $P(-0.38 \leq Z \leq 1.25)$

Example

(e) Find the $99^{th}$ percentile of $Z$

Critical Values of the Standard Normal RV

We denote by $z_{\alpha}$ the $100(1-\alpha)^{th}$ percentile
Example

(g) Find $z_{0.05}$
(h) Find $z_{0.01}$

Nonstandard Normal RV

If $X$ is a normal rv with parameters $\mu$ and $\sigma$, then

$$Z = \frac{X - \mu}{\sigma}$$

is a standard normal rv.

Example

Reaction time for an in-traffic response to a brake signal from standard brake lights can be modeled with a normal distribution having mean value 1.25 seconds and standard deviation of 0.46 seconds.

What is the probability that reaction time is between 1 and 1.75 seconds?

Example

Suppose the IQ among a certain population is a normal rv with mean 100 and standard deviation 15.

Determine a value $c$ such that a randomly selected person has IQ at least $c$ with probability 0.3.

Example

The breakdown voltage of a randomly chosen diode of a particular type is known to be a normal rv with mean value $\mu$ and standard deviation $\sigma$.

- What is the probability that a diode’s breakdown voltage is within $\sigma$ of $\mu$?
- What is the probability that a diode’s breakdown voltage is within $k\sigma$ of $\mu$?

Approximating the Binomial RV

If $X$ is a binomial rv such that $np \geq 10$ and $n(1-p) \geq 10$, then we can approximate $X$ with $Z$:

$$P(X \leq x) = \Phi \left( \frac{x + 0.5 - np}{\sqrt{np(1-p)}} \right)$$
Example

Suppose that 25% of all licensed drivers in a particular state do not have insurance. Let X be the number of uninsured drivers in a random sample of size 50.

- Find \( P(X \leq 10) \) approximately
- Find \( P(5 \leq X \leq 15) \) approximately