Chapter 4
Continuous Random Variables

4.1 Probability Density Functions
4.2 Cumulative Distribution Functions and Expected Values
4.3 The Normal Distribution
4.4 The Exponential and Gamma Distributions
4.5 Other Continuous Distributions

Weibull RV

The Weibull rv $X$ with parameters $\alpha>0$ and $\beta>0$ is the rv with the pdf (for $x\geq 0$)

$$f(x; \alpha, \beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}$$

Expectation and Variance of the Weibull RV

If $X$ is a Weibull rv with parameters $\alpha$ and $\beta$, then

$$E(X) = \beta \Gamma(1+\alpha^{-1})$$
$$V(X) = \beta^2 \left\{ \Gamma(1+2\alpha^{-1})-\Gamma(1+\alpha^{-1})^2 \right\}$$
$$F(x; \alpha, \beta) = 1-e^{-(x/\beta)\alpha} \text{ for } x\geq 0$$

Example

Let $X$ denote the amount of NOx emission (g/gal) from a randomly selected four-stroke engine of a certain type. Suppose that $X$ has a Weibull distribution with $\alpha=2$ and $\beta=10$.

What is the probability that $X$ is
- at most 20?
- more than 40?

Lognormal RV

A nonnegative rv $X$ is said to be a lognormal rv with parameters $\mu$ and $\sigma>0$ if $\ln(X)$ is a normal rv with parameters $\mu$ and $\sigma$.

Expectation and Variance of the Lognormal RV

If $X$ is a lognormal rv with parameters $\mu$ and $\sigma$, then

$$E(X) = e^{\mu+\sigma^2/2}$$
$$V(X) = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$$
Example

In 6 months from today, a stock price is assumed to have a lognormal distribution with parameters $\mu=3.759$ and $\sigma=0.141$.

- Find the mean and the variance of the stock price
- Find an interval such that the probability that the stock price lies in that interval is 0.95