Chapter 5
Joint Probability Distributions

5.1 Jointly Distributed Random Variables
5.2 Expected Values, Covariance, and Correlation
5.3 Statistics and Their Distributions
5.4 The Distribution of the Sample Mean
5.5 The Distribution of a Linear Combination

Joint pmf

The joint probability mass function of two discrete rvs X and Y is defined for all numbers x and y by
\[ p(x,y) = P(X=x \text{ and } Y=y) \]

Example

We throw two dice. Let X and Y denote the numbers of “3” and “4”, respectively.

Find the joint pmf of X and Y.

Marginal pmfs

Let X and Y be two discrete rvs with joint pmf \( p(x,y) \). The marginal probability mass functions of X and Y are defined for all numbers x and y by
\[ p_x(x) = \sum_y p(x,y) \text{ and } p_y(y) = \sum_x p(x,y). \]

Example

We throw two dice. Let X and Y denote the numbers of “3” and “4”, respectively.

Find the marginal pmfs of X and Y.

Independence

Two discrete rvs X and Y with joint pmf \( p(x,y) \) and marginal pmfs \( p_x(x) \) and \( p_y(y) \) are called independent if for every pair of x and y values, we have
\[ p(x,y) = p_x(x)p_y(y). \]
Example

We throw two dice. Let $X$ and $Y$ denote the numbers of “3” and “4”, respectively.
Are $X$ and $Y$ independent?

Conditional pmf

Let $X$ and $Y$ be two discrete rvs with joint pmf $p(x,y)$ and marginal pmfs $p_X(x)$ and $p_Y(y)$. Then the conditional pmf of $Y$ given $X=x$ is defined by $p_{Y|X}(y|x)=p(x,y)/p_X(x)$.

Example

We throw two dice. Let $X$ and $Y$ denote the numbers of “3” and “4”, respectively.
For each value of $x$, calculate the conditional pmf of $Y$ when $X=x$. 