Chapter 7
Confidence Intervals

7.1 Basic Properties of Confidence Intervals
7.2 Large Sample Intervals for a Population Mean and Proportion
7.3 Intervals Based on a Normal Population
7.4 Intervals for the Variance of a Normal Population

Large-Sample CI
After observing \( x_1, x_2, ..., x_n \) from any population with known \( \sigma \), if \( n \) is large (\( n > 40 \)), the interval

\[
\left( X - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, X + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)
\]

is a large-sample 100(1-\( \alpha \))% CI for \( \mu \).

If \( \sigma \) is unknown, replace it by \( s \).

Example
The following are observations of breakdown voltage (kV) of a particular circuit under certain circumstances:

Find a 95% confidence interval for \( \mu \).

Example
The data of a random sample of size 100 from a population with known variance \( \sigma^2 = 20 \) yield a sample mean of 101.5.

(a) Find a 95% confidence interval for \( \mu \).
(b) How large should \( n \) be so that the 99% CI for \( \mu \) has length at most 0.1?
(c) Find a one-sided 95% CI for \( \mu \).

Example
Neither the mean nor the variance of an rv for the weight (in gram) of packages filled by a certain machine is known. A random sample of size 100 gave a sample mean of 985 and a sample standard deviation of 4.5.

The producer wants to give a lower bound for \( \mu \) which is met with 99%. Find such a lower bound.
Example

In 48 trials in a particular laboratory, 16 resulted in ignition of a particular type of substrate by a lighted cigarette.

(a) Find a 95% confidence interval for the proportion $p$ of all such trials that would result in ignition.

(b) How large should $n$ be such that the 95% CI for $p$ is no wider than 0.1?

Example

During a certain year and in a certain state, 113,398 children are born, 58,497 of which are male.

Find a 99% confidence interval for the probability that a newly born child is a boy.