Chapter 7

Confidence Intervals

7.1 Basic Properties of Confidence Intervals
7.2 Large Sample Intervals for a Population Mean and Proportion
7.3 Intervals Based on a Normal Population
7.4 Intervals for the Variance of a Normal Population

χ² Distribution

If \( X_1, X_2, ..., X_n \) are a random sample from a normal population, then the rv

\[
\frac{(n-1)S^2}{\sigma^2}
\]

is a so-called χ² rv with \( n-1 \) df.

Critical Values of the χ² RV

We denote by \( \chi^2_{a,v} \) the 100(1-\( α \))th percentile of the χ² rv with \( v \) df

Confidence Interval for \( \sigma^2 \)

After observing \( x_1, x_2, ..., x_n \) from a normal population with unknown \( \mu \) and \( \sigma \), the interval

\[
\left( \frac{(n-1)s^2}{\chi^2_{a/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-a/2,n-1}} \right)
\]

is a 100(1-\( α \))% confidence interval (CI) for \( \sigma^2 \).

Example

The following are observations of breakdown voltage of electrically stressed circuits (from a normal population):

1470, 1510, 1690, 1740, 1900, 2000, 2030, 2100, 2190, 2200, 2290, 2380, 2390, 2480, 2500, 2580, 2700

Find a 95% confidence interval for \( \sigma^2 \).

Example

A random sample of size 41 is drawn from a normal population. The sample mean is 981 and the sample variance is 28.7.

(a) Find a two-sided 95% confidence interval for \( \sigma^2 \).
(b) Find both one-sided 95% confidence intervals for \( \sigma^2 \).