Summary for $H_0: p=p_0$

Test statistic value:

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

- $H_a: p > p_0$ reject when $z \geq z_{\alpha}$ (upper-tailed test)
- $H_a: p < p_0$ reject when $z \leq -z_{\alpha}$ (lower-tailed test)
- $H_a: p \neq p_0$ reject when $z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$ (two-tailed test)

Use this when $np_0 \geq 10$ and $n(1-p_0) \geq 10$.

Example

The producer of a drug claims that their company’s drug cures a certain disease with probability 0.9. A hospital uses this drug with 400 patients. 340 of them were cured.

Can we reject the producer’s claim with $\alpha=0.01$?

Example

A package-delivery service advertises that at least 90% of all packages brought to its office by 9 a.m. for delivery in the same city are delivered by noon that day. Consider $H_0: p=0.9$ vs $H_a: p<0.9$.

a. If only 80% of the packages are delivered as advertised, how likely is it that a level 0.01 test based on 225 packages will detect such a departure from $H_0$?

b. What should the sample size be to ensure that $\beta(0.8)=0.01$?