Chapter 4

Continuous Random Variables

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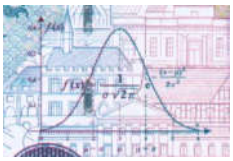
Normal RV



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Normal RV

The normal (or Gaussian) rv X with parameters μ and σ>0 is the rv with the pdf

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

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Expectation and Variance of the normal RV

If X is a normal rv with parameters μ and σ, then

E(X)=μ

V(X)=σ²

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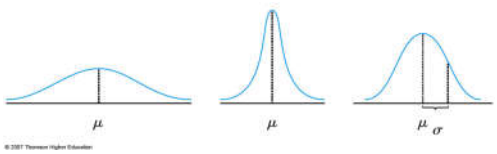
Normal RV



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Standard Normal RV

The standard normal (or Gaussian) rv Z is the normal rv with μ=0 and σ=1 and the pdf

$$f(z;0,1)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

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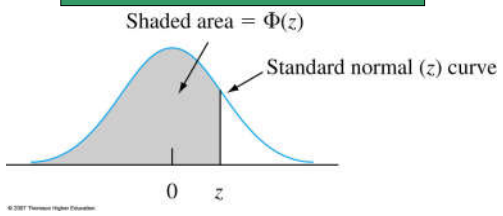
Standard Normal RV



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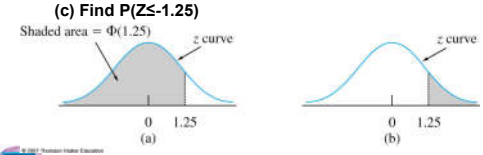
Example

(a) Find P(Z≤1.25)

(b) Find P(Z>1.25)

(c) Find P(Z≤-1.25)



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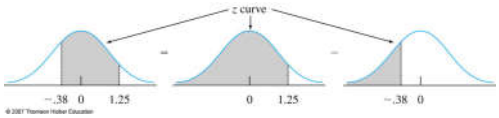
Example

(d) Find P(-0.38≤Z≤1.25)



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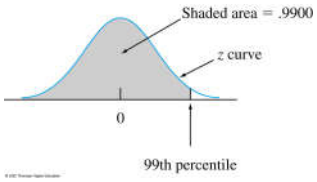
Example

(e) Find the 99th percentile of Z



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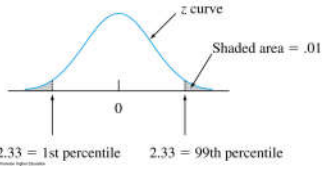
Example

(f) Find the 1st percentile of Z



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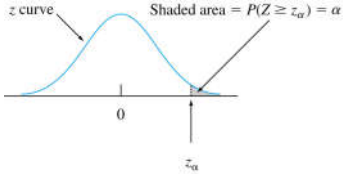
Critical Values of the Standard Normal RV

We denote by z_α the 100(1-α)th percentile



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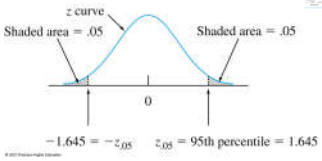
Example

(g) Find $z_{0.05}$

(h) Find $z_{0.01}$



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Nonstandard Normal RV

If X is a normal rv with parameters μ and σ, then

Z=(X-μ)/σ

is a standard normal rv.

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Example

Reaction time for an in-traffic response to a brake signal from standard brake lights can be modeled with a normal distribution having mean value 1.25 seconds and standard deviation of 0.46 seconds.

What is the probability that reaction time is between 1 and 1.75 seconds?

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Example

Suppose the IQ among a certain population is a normal rv with mean 100 and standard deviation 15.

Determine a value c such that a randomly selected person has IQ at least c with probability 0.3.

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Example

The breakdown voltage of a randomly chosen diode of a particular type is known to be a normal rv with mean value μ and standard deviation σ.

- What is the probability that a diode's breakdown voltage is within σ of μ?

- What is the probability that a diode's breakdown voltage is within kσ of μ?

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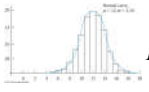
Approximating the Binomial RV

If X is a binomial rv such that np≥10 and n(1-p)≥10, then we can approximate X with Z:



$$P(X \le x) = \Phi\left(\frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

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Example

Suppose that 25% of all licensed drivers in a particular state do not have insurance. Let X be the number of uninsured drivers in a random sample of size 50.

- **Find P(X≤10) approximately**
- **Find P(5≤X≤15) approximately**