

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

## Chapter 7

### Confidence Intervals

- 7.1 Basic Properties of Confidence Intervals
- 7.2 Large Sample Intervals for a Population Mean and Proportion**
- 7.3 Intervals Based on a Normal Population
- 7.4 Intervals for the Variance of a Normal Population

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## Large-Sample CI

After observing  $x_1, x_2, \dots, x_n$  from any population with known  $\sigma$ , if  $n$  is large ( $n > 40$ ), the interval

$$\left( \bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

is a **large-sample**  $100(1-\alpha)\%$  CI for  $\mu$ .  
 If  $\sigma$  is unknown, replace it by  $s$ .

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## Example

The following are observations of breakdown voltage (kV) of a particular circuit under certain circumstances:

62, 50, 53, 57, 41, 53, 55, 61, 59, 64, 50, 53, 64, 62, 50, 68, 54, 55, 57, 50, 55, 50, 56, 55, 46, 55, 53, 54, 52, 47, 47, 55, 57, 48, 63, 57, 57, 55, 53, 59, 53, 52, 50, 55, 60, 50, 56, 58.

Find a **95% confidence interval** for  $\mu$ .

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## Example

The data of a random sample of size 100 from a population with known variance  $\sigma^2=20$  yield a sample mean of 101.5.

- (a) Find a **95% confidence interval** for  $\mu$ .
- (b) How large should  $n$  be so that the **99% CI** for  $\mu$  has length at most 0.1?
- (c) Find a **one-sided 95% CI** for  $\mu$ .

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## Example

Neither the mean nor the variance of an rv for the weight (in gram) of packages filled by a certain machine is known. A random sample of size 100 gave a sample mean of 985 and a sample standard deviation of 4.5.

The producer wants to give a lower bound for  $\mu$  which is met with **99%**. Find such a lower bound.

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## Large-Sample CI

A **large-sample**  $100(1-\alpha)\%$  CI for a population proportion  $p$  is given by

$$\left( \frac{\hat{p} + \frac{z_{\alpha/2}^2}{2n} - z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n} + \frac{z_{\alpha/2}^2}{4n^2}}}{1 + \frac{z_{\alpha/2}^2}{n}}, \frac{\hat{p} + \frac{z_{\alpha/2}^2}{2n} + z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n} + \frac{z_{\alpha/2}^2}{4n^2}}}{1 + \frac{z_{\alpha/2}^2}{n}} \right)$$

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### Example

In 48 trials in a particular laboratory, 16 resulted in ignition of a particular type of substrate by a lighted cigarette.

Find a 95% confidence interval for the proportion  $p$  of all such trials that would result in ignition.



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### Example

During a certain year and in a certain state, 113,398 children are born, 58,497 of which are male.

Find a 99% confidence interval for the probability that a newly born child is a boy.



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