Chapter 7

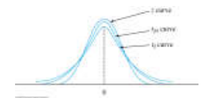
Confidence Intervals

266

t Distribution

If $X_1, X_2, \ldots, X_n$ are a random sample from a normal population, then the rv

$$T = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

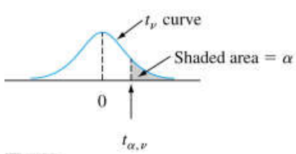


is a so-called t rv with n-1 degrees of freedom (df).

267

Critical Values of the t RV

We denote by $t_{\alpha,\nu}$ the 100(1-α)th percentile of the t rv with ν df



268

Confidence Interval

After observing $x_1, x_2, \ldots, x_n$ from a normal population with unknown σ, the interval

$$\left(\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right)$$

is a 100(1-α)% confidence interval (CI) for μ.

269

One-sided Confidence Intervals

After observing $x_1, x_2, \ldots, x_n$ from a normal population with unknown σ, the intervals

$$\left(-\infty, \bar{x} + t_{\alpha,n-1}\frac{s}{\sqrt{n}}\right) \qquad \left(\bar{x} - t_{\alpha,n-1}\frac{s}{\sqrt{n}}, \infty\right)$$

are two one-sided 100(1-α)% confidence intervals for μ.

270

Example

Consider the following observations on modulus of elasticity (MPa) of Scotch pine lumber specimens at certain conditions:

10,490	16,620	17,300	15,480	12,970	17,260	13,400
13,900	13,630	13,260	14,370	11,700	15,470	17,840
14,070	14,760					

Under normality, find a 95% confidence interval for μ.

271

Example

A psychologist tests 51 people on their reaction times to a certain signal. We assume that this random variable is normal with unknown variance. Suppose the 51 measurements result in a mean sample of 0.8 seconds and a sample variance of 0.04.

(a) Find a 95% confidence interval for μ.

(b) Find both one-sided 95% confidence intervals for μ.