

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

## Chapter 7

### Confidence Intervals

- 7.1 Basic Properties of Confidence Intervals
- 7.2 Large Sample Intervals for a Population Mean and Proportion
- 7.3 Intervals Based on a Normal Population**
- 7.4 Intervals for the Variance of a Normal Population

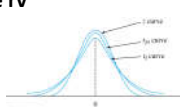
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## t Distribution

If  $X_1, X_2, \dots, X_n$  are a random sample from a normal population, then the rv

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$


is a so-called **t rv** with  $n-1$  **degrees of freedom (df)**.

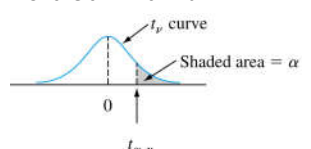
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## Critical Values of the t RV

We denote by  $t_{\alpha, v}$  the  $100(1-\alpha)$ th percentile of the t rv with  $v$  df



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## Confidence Interval

After observing  $x_1, x_2, \dots, x_n$  from a normal population with unknown  $\sigma$ , the interval

$$\left( \bar{x} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}, \bar{x} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \right)$$

is a **100(1- $\alpha$ )% confidence interval (CI)** for  $\mu$ .

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## One-sided Confidence Intervals

After observing  $x_1, x_2, \dots, x_n$  from a normal population with unknown  $\sigma$ , the intervals

$$\left( -\infty, \bar{x} + t_{\alpha, n-1} \frac{S}{\sqrt{n}} \right) \quad \left( \bar{x} - t_{\alpha, n-1} \frac{S}{\sqrt{n}}, \infty \right)$$

are two **one-sided 100(1- $\alpha$ )% confidence intervals** for  $\mu$ .

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## Example

Consider the following observations on modulus of elasticity (MPa) of Scotch pine lumber specimens at certain conditions:

10,490	16,620	17,300	15,480	12,970	17,260	13,400
13,900	13,630	13,260	14,370	11,700	15,470	17,840
14,070	14,760					

Under normality, find a **95% confidence interval** for  $\mu$ .

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### Example

A psychologist tests 51 people on their reaction times to a certain signal. We assume that this random variable is normal with unknown variance. Suppose the 51 measurements result in a mean sample of 0.8 seconds and a sample variance of 0.04.

- (a) Find a 95% confidence interval for  $\mu$ .
- (b) Find both one-sided 95% confidence intervals for  $\mu$ .

