

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 7

Confidence Intervals

- 7.1 Basic Properties of Confidence Intervals
- 7.2 Large Sample Intervals for a Population Mean and Proportion
- 7.3 Intervals Based on a Normal Population
- 7.4 Intervals for the Variance of a Normal Population**

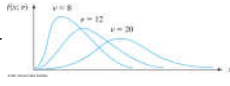
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χ^2 Distribution

If X_1, X_2, \dots, X_n are a random sample from a normal population, then the rv

$$\frac{(n-1)S^2}{\sigma^2}$$


is a so-called χ^2 rv with $n-1$ df.

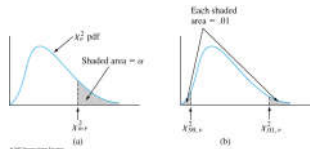
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Critical Values of the χ^2 RV

We denote by $\chi^2_{\alpha, v}$ the $100(1-\alpha)$ th percentile of the χ^2 rv with v df



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Confidence Interval for σ^2

After observing x_1, x_2, \dots, x_n from a normal population with unknown μ and σ , the interval

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}} \right)$$

is a **100(1- α)% confidence interval (CI)** for σ^2 .

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Example

The following are observations of breakdown voltage of electrically stressed circuits (from a normal population):

1470, 1510, 1690, 1740, 1900, 2000,
2030, 2100, 2190, 2200, 2290, 2380,
2390, 2480, 2500, 2580, 2700

Find a **95% confidence interval** for σ^2 .

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Example

A random sample of size 41 is drawn from a normal population. The sample mean is 981 and the sample variance is 28.7.

- (a) Find a **two-sided 95% confidence interval** for σ^2 .
- (b) Find both **one-sided 95% confidence intervals** for σ^2 .

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