Time-Dependent Reliability Analysis by a Sampling Approach to Extreme Values of Stochastic Processes

Zhen Hu
Xiaoping Du
Missouri University of Science and Technology
Outline

• Time-dependent reliability
• A sampling approach to extreme values of stochastic processes
• Reliability analysis
• Examples
• Conclusions
• Future work
Time-Dependent Reliability

• Limit-state functions change with time
  \[ G = g(X, Y(t), t) \]

• Reliability
  \[ R(0, t_s) = \Pr \{ g(X, Y(t)) < 0, \text{ for any } t \in [0, t_s] \} \]

• Reliability is therefore time dependent
  – Probability of success over a period of time
  \[ Y: \text{ A stochastic process} \]
  \[ X: \text{ Random variables} \]
Examples

- **Mechanisms**
  - G-function: $G = g(X, t)$
  - Defined over $[0, t_s]$ or $[\theta_0, \theta_s]$

- **Hydrokinetic turbine**
  - G-function: $G = g(X, Y(t), t)$
  - Stochastic process
    - Water flow velocity
Challenges

• We need the distribution of extreme values of $g(\cdot)$ over $[0,t_s]$.  
  \[ R(0, t_s) = \Pr \{ g(X, Y(t)) < 0, \text{ for any } t \in [0, t_s] \} \]

• Monte Carlo simulation is too expensive.

• The most commonly used method is inaccurate
  – Upcrossing rate method
New Methodology

• Limit-state function \( G = g(X, Y(t)) \)

• Decompose \( Y \) into \( Y_R \) and \( Y_S \)
  – \( Y_R \): Generalized strength variables
  – \( Y_S \): Generalized stress variables

• Worst case over \([0, t_s]\) with
  – Minimum \( Y_R \) and maximum \( Y_S \)

• Time-dependent \(\rightarrow\) Time-independent

\[
p_f(0, t_s) = \Pr\left\{ g(X, Y(t)) > 0, \exists t \in [0, t_s] \right\} = \Pr\left\{ \max g(X, Y_R(t), Y_S(t)) > 0, t \in [0, t_s] \right\} = \Pr\left\{ g(X, Y_R^{\min}, Y_S^{\max}) > 0 \right\}
\]
Task and Approach

\[ p_f(0, t_s) = \Pr \left\{ g(X, Y_{R \min}, Y_{S \max}) > 0 \right\} \]

• Task: find distributions of \( Y_{R \min} \) and \( Y_{S \max} \).

• Approach
  – Use Monte Carlo simulation (MCS)
  – Sample on \( Y_R \) and \( Y_S \) over \([0, t_s]\)
  – Obtain samples of \( Y_{R \min} \) and \( Y_{S \max} \).
  – It will not call the g-function

• Then time-independent analysis
  – FORM, SORM, etc.
Sampling Approach

- Expansion Optimal Linear Estimation (EOLE) is used to generate samples for $Y$
- Saddlepoint Approximation (SPA) is employed to approximate the CDF of $Y^\text{min}_R$ and $Y^\text{max}_S$
- SPA maintains the robustness of reliability analysis
Procedure

• Identify $Y_R$ and $Y_S$
• Find distributions of the extreme values of $Y_R$ and $Y_S$ by sampling
• Perform time-invariant reliability analysis
Example: Hydrokinetic Turbine Blades

- Random variables
  \[ X = [l_1, t_1, t_2, \varepsilon_{allow}] \]

- Generalized stress variables
  \[ Y_s = v(t) \]
Limit-State Function

Bending moment

\[ M_{flap} = \frac{1}{2} \rho v(t)^2 C_m \]

Limit-state function

\[ g = \varepsilon_{allow} - \frac{M_{flap} t_1}{EI} = \varepsilon_{allow} - \frac{\rho v(t)^2 C_m t_1}{2EI} \]

\[ I = \frac{1}{12} l_1 ( (2t_1)^3 - (2t_2)^3 ) = \frac{2}{3} l_1 (t_1^3 - t_2^3) \]

\( v(t) \) is a non-stationary stochastic process

Mean and standard deviation are functions of time
Results: Accuracy

UC: Up-crossing rate method

Maximum river velocity
## Results: Efficiency

### Number of function calls

<table>
<thead>
<tr>
<th>Time interval (months)</th>
<th>UC</th>
<th>Proposed</th>
<th>MCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 4]</td>
<td>2522</td>
<td>102</td>
<td>$10^6$</td>
</tr>
<tr>
<td>[0, 5]</td>
<td>6539</td>
<td>85</td>
<td>$10^6$</td>
</tr>
<tr>
<td>[0, 6]</td>
<td>6761</td>
<td>121</td>
<td>$10^6$</td>
</tr>
<tr>
<td>[0, 7]</td>
<td>11500</td>
<td>98</td>
<td>$10^6$</td>
</tr>
<tr>
<td>[0, 8]</td>
<td>6678</td>
<td>98</td>
<td>$10^6$</td>
</tr>
<tr>
<td>[0, 9]</td>
<td>19399</td>
<td>98</td>
<td>$10^6$</td>
</tr>
</tbody>
</table>
Conclusions

- The accuracy of the proposed method is good
- The proposed method is efficient
- Applicable for problems with non-stationary stochastic loading

Limitations

- \( G = g(X, Y(t)) \)
- \( Y \) can be decomposed into \( Y_R \) and \( Y_S \)
Future Work

• We are working on more advanced methodologies
  – General problems $G=g(\mathbf{X}, Y(t), t)$
  – Time-dependent system reliability

• Design optimization with time-dependent uncertainties
Acknowledgement

• ONR N000141010923
• The Intelligent Systems Center at the Missouri University of Science and Technology.