Extreme Value Metamodelling for System Reliability with Time-Dependent Functions

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Outline

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  • Kriging model
  • Mixed Efficient Global Optimization (mEGO)

• Mixed System EGO (mSEGO)

• Example

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Objective

- Objective
  Develop a new time-dependent system reliability method

A series system                A parallel system   Two parallel subsystems in series
Time-Dependent System Reliability

The limit-state function of failure mode $i$

$$Y_i = g_i(X,t), t \in [t_0, t_s]$$

$Y_i > 0$ leads to a failure.

Probability of failure

$$p_f^i(t_0, t_s) = \Pr\{g_i(X, t) > 0, \exists t \in [t_0, t_s]\}$$

For a series system, the system probability of failure

$$p_f^s(t_0, t_s) = \Pr\{g_1(X, t) > 0 \cup g_2(X, t) > 0 \cup \cdots \cup g_n(X, t) > 0, \exists t \in [t_0, t_s]\}$$
Extreme Values

\[ p^S_f(t_0, t_s) = \Pr\{ \max_{i=1,2,\ldots,n} g_i(X, t) > 0, \exists t \in [t_0, t_s] \} \]

\[ = \Pr\{ \max_{t \in [t_0, t_s]} \max_{i=1,2,\ldots,n} g_i(X, t) > 0 \} \]

Let extreme values be

\[ Y_i^{\max} = g_i^{\max}(X) = \max_{t \in [t_0, t_s]} g_i(X, t) \]

Therefore

\[ p^S_f(t_0, t_s) = \Pr\left\{ \max_{i=1,2,\ldots,n} \left( Y_i^{\max} \right) > 0 \right\} \]

We create surrogate models for \( Y_i^{\max} \)
Surrogate Modeling-Kriging Model

- Kriging prediction and variance for $g(x)$

$$\hat{y} = \hat{g}(x) \sim N(\mu_g(x), \sigma^2_g(x))$$

- Our problem: build $\hat{Y}_i^{\text{max}}$ for $g_i(X,t), (i = 1, 2, \cdots, n)$

- Solution: mixed Efficient Global Optimization\textsuperscript{[1]} (mEGO)

**mEGO method has two major advantages:**

1) Sample variables $X$ and $t$ simultaneously.

2) Use AK-MCS\textsuperscript{[2]} to improve efficiency.


mEGO Procedures

- For each point of $x_{MCS}$, find the global maximum response over $[t_0, t_s]$ using $El$.
- $x^*_{new}$ is a point from $x_{MCS}$ with minimum $U$ value.

$U$-function defined by AK-MCS

$$U_{\hat{g}^\text{max}}(x) = \frac{|\mu_{\hat{g}^\text{max}}(x)|}{\sigma_{\hat{g}^\text{max}}(x)}$$

Indicate the accuracy of Kriging model at the limit-state.

mEGO is only for component
For a series system, the composite prediction $\mu^*$ [3] is

$$\mu^*(x) = \max(\hat{g}_i^{\text{max}}(x))$$

High efficiency
- Sample $X$ and $t$ simultaneously
- Check component contributions

Example

Motion outputs

\[ S_i = X_c \cos \theta_i + \sqrt{X_i^2 - (X_c \sin \theta_i)^2} \]

Required motion outputs

\[ S_{R_i} = \mu_c \cos \theta_i + \sqrt{\mu_i^2 - (\mu_c \sin \theta_i)^2} \]

The motion errors

\[ \Delta S_i = |S_{R_i} - S_i| \]

Component probabilities of failure

\[ Y_i = g_i(X, t) = \left( \frac{(X_c - \mu_c) \cos \theta_i + \sqrt{X_i^2 - (X_c \sin \theta_i)^2} - \sqrt{\mu_i^2 - (\mu_c \sin \theta_i)^2}}{\varepsilon_i} \right) \]

Allowable motion errors: \( \varepsilon_i = 4.8, 5.5, 5.2 \text{mm.} \)

System probability of failure

\[ p_f^S = \Pr \{ Y_1^{\max} > 0 \cup Y_2^{\max} > 0 \cup Y_3^{\max} > 0 \} \]
Motion error of mechanism 1 at point (100.5, 150.0) and (100.5, 151.0) mm

Extreme motion error of mechanism 1
Results

Time interval \([0, 2\pi]\) second is divided into 360 time instants. For MCS, \(10^6\) samples are generated at each time instant.

<table>
<thead>
<tr>
<th>Method</th>
<th>(p_f)</th>
<th>Error (%)</th>
<th>Function calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS</td>
<td>1.76E-4</td>
<td>N/A</td>
<td>((3.6, 3.6, 3.6)\times10^8)</td>
</tr>
<tr>
<td>mSEGO</td>
<td>1.71E-4</td>
<td>2.95%</td>
<td>((268, 365, 261))</td>
</tr>
</tbody>
</table>

Conclusions

mSEGO method works well for the following systems:

- Limit-state functions are explicit functions of time.
- No stochastic processes in the input variables.
- Components of system can be in series, parallel, or their combination.
Future Work

- Share the training points and their responses among the components.
- Use adaptive convergence criterion for the $EI$ for the extreme responses.

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