Robust Design

ME 261 Engineering Design
2014 Spring
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Outline

• Introductory Examples
• Definition of Robustness
• Statistics
• How to achieve robustness
• Examples
• Related Issue: Reliability
• Conclusions
An Example: Cantilever Beam

The material strength $S$

The maximum stress $\sigma_{\text{max}} = \frac{6P(l_1 + l_2)}{b_2h_2^2}$

Factor of safety $F_s = \frac{S}{\sigma_{\text{max}}} > 1$

How big is enough?

Reality: everything is uncertain
P=\(\{2001.4, 1531.3, 2534.6, \ldots\}\) kN
S=\(\{120.5, 101.3, 131.2, 170.9, \ldots\}\) MPa
$h_1 = 100 \pm 0.01$ mm
$b_1 = 50 \pm 0.01$ mm, \ldots
Which Design is Robust Under Uncertainty?

Sensitive to variation

Robust

Increased Robustness
Variation (Uncertainty)

- Piece-to-piece variation
- Customer usage and duty cycle
- Human errors
- Model inaccuracy
How Do We Quantify Uncertainty?

- Support we have 100 measurements for $X = L_1$
- $(99.99, 100.08, \ldots, 100.05)$ mm

**Average**

Mean $\bar{X} = \frac{1}{100} \sum_{i=1}^{100} x_i = 99.96$

**Dispersion**

Standard deviation $\sigma = \sqrt{\frac{1}{100-1} \sum_{i=1}^{n} (x_i - \bar{X})^2}$

**Histogram**

**Probability density (distribution)**
Probability Distribution

- Mean – average
- Standard deviation (std) – dispersion around the average or amount of variation

- Two designs with two performance variables
  - $\bar{X}_1 = \bar{X}_2$, $\sigma_2 > \sigma_1$
  - Designs 1 and 2 have the same average performance
  - But the variation of Design 2 is higher.
Concept of Robustness

Big or dangerous?
Robustness

• The robustness of a product is the ability that its performances are not affected by the uncertain inputs or environment conditions (noises).

• A robust product can work under large uncertainties.
TV Example

• In 1970s, Americans showed a preference for television sets made by Sony-Japan over those made by Sony-USA.

• The color density was a major performance.
  – Target ± tolerance = m ± 5.

• Sony-Japan sets: 0.3% defective sets (outside the tolerance limits)

• Sony-USA sets: virtually NO sets outside the tolerance limits.

• Why?

(Phadke, 1989)
TV Example

- Sony-USA: Uniform distribution, $\sigma = 2.89$
- Sony-Japan: Normal distribution, $\sigma = 1.67$, and most of the sets are grade A.

(Phadke, 1989)
How to Evaluate Robustness?

- Traditional design
  \( m \) – target, \( \Delta \) – tolerance

- Robust design
- Taguchi’s quality loss (\$)
  \[ L = k(Y - m)^2 \]
  \( k \) - constant
Expected Quality Loss

• Expected (average) quality loss

\[ E(L) = k [(\bar{Y} - m)^2 + \sigma_Y^2] \]

• Reducing \( E(L) \) will bring the average performance to the target and reducing variation \( \sigma_Y \) simultaneously.
Other Types of Quality Loss

• What we’ve discussed is the “nominal-the-better” type

• The “smaller-the-better” type
  – cost, stress, energy consumption

• The “larger-the-better” type
  – life, reliability, strength, efficiency
Robust Design

Increasing average performance

Decreasing variation

Increasing robustness
Achieve Robustness During Conceptual Design

• Keep uncertainty in mind
• Define the robustness target
• Identify the root causes of variations
• Make design concepts not sensitive to variations in later design stages, manufacturing, and operation.
  – For example, if the fluctuation in temperature is high, use a pair of gears instead of a belt.
Achieve Robustness During Parameter Design

- Performance $Y = g(X_1, X_2, \ldots, X_n)$
- Design variables $\overline{X} = (\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)$
- $X_i (i = 1, 2, \ldots, n)$ are independent
Parameter Design

- Average performance
  \[ \bar{Y} = f(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n) \]

- Taylor expansion series
  \[ Y \approx \bar{Y} + c_1(X_1 - \bar{X}_1) + c_2(X_2 - \bar{X}_2) + \ldots + c_n(X_n - \bar{X}_n) \]

- Partial derivatives
  \[ c_i = \frac{\partial g}{\partial X_i} \text{ at } \bar{X} \quad (i = 1, 2, \ldots, n) \]

- Standard deviation
  \[ \text{Std }\sigma_Y = \sqrt{c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + \ldots + c_n^2\sigma_n^2} \]

- Change \( \bar{X} \) (not reduce \( \sigma_i \)) to reduce (or minimize)
  \[ E(L) = k[(\bar{Y} - m)^2 + \sigma_Y^2] \]
Example: Robust Mechanism Synthesis

- Requirements:
  \[ s = m_1 = 350 \text{ mm when } \theta = 10^\circ \]
  \[ s = m_2 = 250 \text{ mm when } \theta = 60^\circ \]

- Uncertainties in \( a, b, \) and \( e \)
  \[ \sigma_a = 2 \text{ mm}, \sigma_b = 2 \text{ mm}, \sigma_e = 3 \text{ mm} \]

- Design variables
  \( \bar{a}, \bar{b}, \bar{e} \)
Results

\[ s = a \cos \theta + \sqrt{b^2 - (e + a \sin \theta)^2} \]

Transmission angle > 45°

Minimize \( \sum_{i=1}^{2} k \left[ (\bar{s}_i - m_i)^2 + \sigma^2_{si} \right] \)

<table>
<thead>
<tr>
<th>Method</th>
<th>Deterministic Design</th>
<th>Robust Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{a} ) (mm)</td>
<td>119.6</td>
<td>136.6</td>
</tr>
<tr>
<td>( \bar{b} ) (mm)</td>
<td>241.3</td>
<td>216.8</td>
</tr>
<tr>
<td>( \bar{e} ) (mm)</td>
<td>45.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( \bar{s} ) (( \theta = 10^\circ )) (mm)</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>( \bar{s} ) (( \theta = 60^\circ )) (mm)</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>( \sigma ) (( \theta = 10^\circ )) (mm)</td>
<td>2.9</td>
<td>2.8</td>
</tr>
<tr>
<td>( \sigma ) (( \theta = 60^\circ )) (mm)</td>
<td>3.5</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Example 2 - Piston Engine Robust Design

\[
\begin{align*}
\min_{\mu_x} & \quad w_1 \mu_f + w_2 \sigma_f \\
\text{s.t.} & \quad P\{G < 7\} \geq 0.99
\end{align*}
\]

\(f – \text{Slap noise} \quad G - \text{Friction}\)

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Optimal</th>
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</thead>
<tbody>
<tr>
<td>Mean of (f)</td>
<td>54.5 dB</td>
<td>54.2 dB</td>
</tr>
<tr>
<td>Std of (f)</td>
<td>2.04 dB</td>
<td>0.76 dB</td>
</tr>
<tr>
<td>Prob</td>
<td>0.65</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Related Issue: Reliability (R)

• The ability (probability) of a product performing its intended function

\[ R = \Pr\{\text{design points falling into safe region}\} \]

\[ R = \Pr\{\text{strength} > \text{stress}\} \]

\[ R = \Pr\{\text{factor of safety} > 1\} \]
Reliability vs. Robustness

Everyday fluctuations around the mean – deterioration, degradation, quality loss

Extreme events at tails – failure, catastrophe

- Reliability issue
  - Motherboard failure
  - Broken hard disk
- Robustness issue
  - Overheating
  - Noise
How to Design for Reliability?

• Conceptual design
  - Failure mode and effects analysis (FMEA)

• Parameter design
  - Reliability-based design
Conclusions

• Robust design -> insensitivity to uncertainties
  – Insensitive to material variations -> use of low grade materials and components -> low material cost
  – Insensitive to manufacturing variations -> no tightened tolerances -> low manufacturing and labor cost
  – Insensitive to variations in operation environment -> low operation cost

• Robust design -> increased performance, quality, and reliability at reduced cost

• Robustness and reliability can be built into products during early stages of design.
More Information

• Visit the website of Engineering Uncertainty Repository at http://www.mst.edu/~dux/repository.

• Contact me for any help in Toomey Hall 290D or at dux@mst.edu.