Figure 1.1  Astrophysical $N$-body simulation by Scott Linssen (undergraduate University of North Carolina at Charlotte [UNCC] student).
Figure 1.2  Conventional computer having a single processor and memory.
Figure 1.3 Traditional shared memory multiprocessor model.
Figure 1.4  Message-passing multiprocessor model (multicomputer).
Figure 1.5  Shared memory multiprocessor implementation.
Figure 1.6 MPMD structure.
Figure 1.7  Static link multicomputer.
Figure 1.8 Node with a switch for internode message transfers.
Figure 1.9  A link between two nodes with separate wires in each direction.
Figure 1.10  Ring.
Figure 1.11  Two-dimensional array (mesh).
Figure 1.12  Tree structure.
Figure 1.13  Three-dimensional hypercube.
Figure 1.14 Four-dimensional hypercube.
Figure 1.15  Embedding a ring onto a torus.
Figure 1.16  Embedding a mesh into a hypercube.
Figure 1.17  Embedding a tree into a mesh.
Figure 1.18 Distribution of flits.
Figure 1.19 A signaling method between processors for wormhole routing (Ni and McKinley, 1993).
Figure 1.20 Network delay characteristics.
Figure 1.21  Deadlock in store-and-forward networks.
Figure 1.22  Multiple virtual channels mapped onto a single physical channel.
**Figure 1.23** Ethernet-type single wire network.

![Diagram of Ethernet network with workstations and file server connected through Ethernet]
<table>
<thead>
<tr>
<th>Frame check sequence (32 bits)</th>
<th>Data (variable)</th>
<th>Type (16 bits)</th>
<th>Source address (48 bits)</th>
<th>Destination address (48 bits)</th>
<th>Preamble (64 bits)</th>
</tr>
</thead>
</table>

**Figure 1.24** Ethernet frame format.
Figure 1.25  Network of workstations connected via a ring.
Figure 1.26  Star connected network.
Figure 1.27 Overlapping connectivity Ethernets.
Time

Process 1
Process 2
Process 3
Process 4

Waiting to send a message  Message  Time

Computing
Slope indicating time to send message

Figure 1.28  Space-time diagram of a message-passing program.
Figure 1.29  Parallelizing sequential problem — Amdahl’s law.
Figure 1.30  (a) Speedup against number of processors. (b) Speedup against serial fraction, $f$. 
Figure 2.1 Single program, multiple data operation.
Figure 2.2  Spawning a process.
Figure 2.3  Passing a message between processes using `send()` and `recv()` library calls.
Figure 2.4 Synchronous send() and recv() library calls using a three-way protocol.
Figure 2.5 Using a message buffer.
Figure 2.6  Broadcast operation.
Figure 2.7 Scatter operation.
Figure 2.8  Gather operation.
Figure 2.9  Reduce operation (addition).
Figure 2.10  Message passing between workstations using PVM.
Figure 2.11  Multiple processes allocated to each processor (workstation).
Figure 2.12 pvm_psend() and pvm_precv() system calls.
Figure 2.13  PVM packing messages, sending, and unpacking.
```c
#include <stdio.h>
#include <stdlib.h>
#include <pvm3.h>
#define SLAVE "spsum"
#define PROC 10
#define NELEM 1000

main() {
    int mytid, tids[PROC];
    int n = NELEM, nproc = PROC;
    int no, i, who, msgtype;
    int data[NELEM], result[PROC], tot = 0;
    char fn[255];
    FILE *fp;
    mytid = pvm_mytid(); /* Enroll in PVM */

    /* Start Slave Tasks */
    no = pvm_spawn(SLAVE, (char**)0, 0, "", nproc, tids);
    if (no < nproc) {
        printf("Trouble spawning slaves \n");
        for (i = 0; i < no; i++) pvm_kill(tids[i]);
        pvm_exit(); exit(1);
    }

    /* Open Input File and Initialize Data */
    strcpy(fn, getenv("HOME"));
    strcat(fn, "/pvm3/src/rand_data.txt");
    if ((fp = fopen(fn, "r")) == NULL) {
        printf("Can't open input file %s\n", fn);
        exit(1);
    }
    for (i = 0; i < n; i++) fscanf(fp, "%d", &data[i]);

    /* Broadcast data To slaves*/
    pvm_initsend(PvmDataDefault);
    msgtype = 0;
    pvm_pkint(&nproc, 1, 1);
    pvm_pkint(tids, nproc, 1);
    pvm_pkint(&n, 1, 1);
    pvm_pkint(data, n, 1);
    pvm_mcast(tids, nproc, msgtag);

    /* Get results from Slaves*/
    msgtype = 5;
    for (i = 0; i < nproc; i++) {
        pvm_recv(-1, msgtype);
        pvm_upkint(&me, 1, 1);
        pvm_upkint(&sum, 1, 1);
        printf("%d from %d\n", result[who], who);
    }

    /* Compute global sum */
    for (i = 0; i < nproc; i++) tot += result[i];
    printf("The total is %d.\n", tot);
    pvm_exit(); /* Program finished. Exit PVM */
}
```

Figure 2.14 Sample PVM program.

Parallel Programming: Techniques and Applications using Networked Workstations and Parallel Computers
Barry Wilkinson and Michael Allen © Prentice Hall, 1998
44
Figure 2.15 Unsafe message passing with libraries.
```c
#include "mpi.h"
#include <stdio.h>
#include <math.h>
#define MAXSIZE 1000

void main(int argc, char *argv)
{
    int myid, numprocs;
    int data[MAXSIZE], i, x, low, high, myresult, result;
    char fn[255];
    char *fp;

    MPI_Init(&argc,&argv);
    MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
    MPI_Comm_rank(MPI_COMM_WORLD,&myid);

    if (myid == 0) { /* Open input file and initialize data */
        strcpy(fn, getenv("HOME"));
        strcat(fn,"/MPI/rand_data.txt");
        if ((fp = fopen(fn,"r")) == NULL) {
            printf("Can't open the input file: %s\n", fn);
            exit(1);
        }
        for(i = 0; i < MAXSIZE; i++) fscanf(fp,"%d", &data[i]);
    }

    /* broadcast data */
    MPI_Bcast(data, MAXSIZE, MPI_INT, 0, MPI_COMM_WORLD);

    /* Add my portion Of data */
    x = n/nproc;
    low = myid * x;
    high = low + x;
    for(i = low; i < high; i++)
        myresult += data[i];
    printf("I got %d from %d\n", myresult, myid);

    /* Compute global sum */
    MPI_Reduce(&myresult, &result, 1, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);
    if (myid == 0) printf("The sum is %d.\n", result);

    MPI_Finalize();
}
```

**Figure 2.16** Sample MPI program.
Figure 2.17  Theoretical communication time.
Figure 2.18 Growth of function $f(x) = 4x^2 + 2x + 12$. 
Figure 2.19  Broadcast in a three-dimensional hypercube.
Figure 2.20  Broadcast as a tree construction.
Figure 2.21  Broadcast in a mesh.
Figure 2.22  Broadcast on an Ethernet network.
Figure 2.23 1-to-$N$ fan-out broadcast.
Figure 2.24 1-to-N fan-out broadcast on a tree structure.
Figure 2.25  Space-time diagram of a parallel program.
Figure 2.26  Program profile.
Figure 3.1 Disconnected computational graph (embarrassingly parallel problem).
Figure 3.2 Practical embarrassingly parallel computational graph with dynamic process creation and the master-slave approach.
Figure 3.3  Partitioning into regions for individual processes.
Figure 3.4 Mandelbrot set.
Figure 3.5  Work pool approach.
Figure 3.6 Counter termination.
Figure 3.7 Computing $\pi$ by a Monte Carlo method.
Figure 3.8  Function being integrated in computing \( \pi \) by a Monte Carlo method.
Figure 3.9 Parallel Monte Carlo integration.
Figure 3.10  Parallel computation of a sequence.
Figure 4.1 Partitioning a sequence of numbers into parts and adding the parts.
Figure 4.2  Tree construction.
Figure 4.3  Dividing a list into parts.
Figure 4.4  Partial summation.
Figure 4.5  Part of a search tree.
Figure 4.6  Quadtree.
Figure 4.7 Dividing an image.
Figure 4.8  Bucket sort.
Figure 4.9 One parallel version of bucket sort.
Figure 4.10 Parallel version of bucket sort.
**Figure 4.11** “All-to-all” broadcast.
“All-to-all”

**Figure 4.12** Effect of “all-to-all” on an array.
Figure 4.13  Numerical integration using rectangles.
Figure 4.14 More accurate numerical integration using rectangles.
Figure 4.15  Numerical integration using the trapezoidal method.
Figure 4.16  Adaptive quadrature construction.
Figure 4.17  Adaptive quadrature with false termination.
Figure 4.18  Clustering distant bodies.
Figure 4.19  Recursive division of two-dimensional space.
Figure 4.20  Orthogonal recursive bisection method.
Figure 4.21 Process diagram for Problem 4-12(b).
Figure 4.22  Bisection method for finding the zero crossing location of a function.
Figure 4.23  Convex hull (Problem 4-22).
Figure 5.1 Pipelined processes.
Figure 5.2  Pipeline for an unfolded loop.
Figure 5.3  Pipeline for a frequency filter.
Figure 5.4  Space-time diagram of a pipeline.
Figure 5.5 Alternative space-time diagram.
Figure 5.6  Pipeline processing 10 data elements.
Figure 5.7  Pipeline processing where information passes to next stage before end of process.
Figure 5.8  Partitioning processes onto processors.
Figure 5.9  Multiprocessor system with a line configuration.
Figure 5.10  Pipelined addition.
Figure 5.11  Pipelined addition numbers with a master process and ring configuration.
Figure 5.12 Pipelined addition of numbers with direct access to slave processes.
Figure 5.13 Steps in insertion sort with five numbers.
Series of numbers $x_{n-1} \ldots x_1 x_0$

$P_0$  Compare

$x_{\text{max}}$  Largest number

$P_1$  Smaller numbers

$P_2$  Next largest number

**Figure 5.14**  Pipeline for sorting using insertion sort.
Figure 5.15  Insertion sort with results returned to the master process using a bidirectional line configuration.
Sorting phase

Returning sorted numbers

2n − 1

n

P4
P3
P2
P1
P0

Figure 5.16  Insertion sort with results returned.
Figure 5.17  Pipeline for sieve of Eratosthenes.
Figure 5.18  Solving an upper triangular set of linear equation using a pipeline.
Figure 5.19  Pipeline processing using back substitution.
\begin{center}
\begin{tabular}{c|c|c|c|c}
$P_0$ & $P_1$ & $P_2$ & $P_3$ & $P_4$ \\
\hline
divide & recv($x_0$) & recv($x_0$) & recv($x_0$) & recv($x_0$) \\
send($x_0$) & send($x_0$) & send($x_0$) & send($x_0$) & send($x_0$) \\
end & multiply/add & multiply/add & multiply/add & multiply/add \\
& recv($x_1$) & recv($x_1$) & recv($x_1$) & recv($x_1$) \\
& send($x_1$) & send($x_1$) & send($x_1$) & send($x_1$) \\
& multiply/add & multiply/add & multiply/add & multiply/add \\
& divide/subtract & divide/subtract & divide/subtract & divide/subtract \\
& send($x_2$) & send($x_2$) & send($x_2$) & send($x_2$) \\
& end & multiply/add & multiply/add & multiply/add \\
& recv($x_3$) & recv($x_3$) & recv($x_3$) & recv($x_3$) \\
& send($x_3$) & send($x_3$) & send($x_3$) & send($x_3$) \\
& multiply/add & multiply/add & multiply/add & multiply/add \\
& divide/subtract & divide/subtract & divide/subtract & divide/subtract \\
& send($x_4$) & send($x_4$) & send($x_4$) & send($x_4$) \\
& end & & & \\
\end{tabular}
\end{center}

**Figure 5.20** Operations in back substitution pipeline.
Figure 5.21  Pipeline for Problem 5-9.
Figure 5.22  Audio histogram display.
Figure 6.1 Processes reaching the barrier at different times.
Processes wait until all reach their barrier call.

Figure 6.2 Library call barriers.
Figure 6.3 Barrier using a centralized counter.
for (i=0; i<n; i++)
    recv(P any);

for (i=0; i<n; i++)
    send(P i);

Master

Slave processes

Arrival phase

Departure phase

Figure 6.4  Barrier implementation in a message-passing system.
Figure 6.5  Tree barrier.
Figure 6.6  Butterfly construction.
Instruction

\[ a[i] = a[i] + k; \]

**Figure 6.7** Data parallel computation.
Figure 6.8  Data parallel prefix sum operation.
Figure 6.9  Convergence rate.
Figure 6.10 Allgather operation.
Figure 6.11  Effects of computation and communication in Jacobi iteration.
Figure 6.12  Heat distribution problem.
Figure 6.13  Natural ordering of heat distribution problem.
Figure 6.14  Message passing for heat distribution problem.
Figure 6.15  Partitioning heat distribution problem.
Figure 6.16  Communication consequences of partitioning.
Figure 6.17  Startup times for block and strip partitions.
Figure 6.18 Configuring array into contiguous rows for each process, with ghost points.
Figure 6.19  Room for Problem 6-14.
Figure 6.20  Road junction for Problem 6-16.
Figure 6.21  Figure for Problem 6-23.
(a) Imperfect load balancing leading to increased execution time

(b) Perfect load balancing

Figure 7.1 Load balancing.
Figure 7.2  Centralized work pool.
Figure 7.3  A distributed work pool.
Figure 7.4  Decentralized work pool.
Figure 7.5  Decentralized selection algorithm requesting tasks between slaves.
Figure 7.6  Load balancing using a pipeline structure.
If buffer empty, make request
Receive task from request

If free, request task
Receive task from request

If buffer full, send task
Request for task

Figure 7.7 Using a communication process in line load balancing.
Figure 7.8  Load balancing using a tree.
Figure 7.9  Termination using message acknowledgments.
Figure 7.10  Ring termination detection algorithm.
Figure 7.11 Process algorithm for local termination.
Figure 7.12 Passing task to previous processes.
Figure 7.13  Tree termination.
Figure 7.14  Climbing a mountain.
Figure 7.15  Graph of mountain climb.
**Figure 7.16** Representing a graph.

(a) Adjacency matrix

<table>
<thead>
<tr>
<th>Source</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>B</td>
<td>∞</td>
<td>8</td>
<td>13</td>
<td>24</td>
<td>51</td>
<td>∞</td>
</tr>
<tr>
<td>C</td>
<td>∞</td>
<td>∞</td>
<td>14</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>D</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>9</td>
<td>∞</td>
<td>∞</td>
</tr>
<tr>
<td>E</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>17</td>
<td>∞</td>
</tr>
<tr>
<td>F</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

(b) Adjacency list

- **Source:** A, B, C, D, E, F
- **Destination:** A, B, C, D, E, F
- **Weight:** NULL
- **Weight:** 10, 8, 13, 24, 51

- **A** (10) → **B** (8) → **D** (13) → **E** (24) → **F** (51)
Figure 7.17 Moore’s shortest-path algorithm.
Figure 7.18  Distributed graph search.
Figure 7.19  Sample maze for Problem 7-9.
**Figure 7.20** Plan of rooms for Problem 7-10.
Figure 7.21  Graph representation for Problem 7-10.
Figure 8.1  Shared memory multiprocessor using a single bus.
**TABLE 8.1** SOME EARLY PARALLEL PROGRAMMING LANGUAGES

<table>
<thead>
<tr>
<th>Language</th>
<th>Originator/date</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concurrent Pascal</td>
<td>Brinch Hansen, 1975&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Extension to Pascal</td>
</tr>
<tr>
<td>Ada</td>
<td>U.S. Dept. of Defense, 1979&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Completely new language</td>
</tr>
<tr>
<td>Modula-P</td>
<td>Bräunl, 1986&lt;sup&gt;c&lt;/sup&gt;</td>
<td>Extension to Modula 2</td>
</tr>
<tr>
<td>C*</td>
<td>Thinking Machines, 1987&lt;sup&gt;d&lt;/sup&gt;</td>
<td>Extension to C for SIMD systems</td>
</tr>
<tr>
<td>Concurrent C</td>
<td>Gehani and Roome, 1989&lt;sup&gt;e&lt;/sup&gt;</td>
<td>Extension to C</td>
</tr>
<tr>
<td>Fortran D</td>
<td>Fox et al., 1990&lt;sup&gt;f&lt;/sup&gt;</td>
<td>Extension to Fortran for data parallel programming</td>
</tr>
</tbody>
</table>

<sup>d</sup> Thinking Machines Corp. (1990), *C* Programming Guide, Version 6, Thinking Machines System Documentation.
Figure 8.2  FORK-JOIN construct.
Figure 8.3  Differences between a process and threads.
Main program

```c
pthread_create(&thread1, NULL, proc1, &arg);

pthread_join(thread1, *status);

proc1(&arg)
```{
```c
{

return(*status);
}
```
}

```
```{thread1

```

Figure 8.4  

pthread_create() and pthread_join().
**Figure 8.5**  Detached threads.
Shared variable, \( x \)

**Process 1**

**Process 2**

**Figure 8.6** Conflict in accessing shared variable.
while (lock == 1) do nothing;
lock = 1;

Critical section

lock = 0;

Figure 8.7 Control of critical sections through busy waiting.
Figure 8.8  Deadlock (deadly embrace).

(a) Two-process deadlock

(b) $n$-process deadlock
Figure 8.9  False sharing in caches.
Figure 8.10 Shared memory locations for Section 8.4.1 program example.
Figure 8.11  Shared memory locations for Section 8.4.2 program example.
TABLE 8.2  LOGIC CIRCUIT DESCRIPTION FOR FIGURE 8.12

<table>
<thead>
<tr>
<th>Gate</th>
<th>Function</th>
<th>Input 1</th>
<th>Input 2</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AND</td>
<td>Test1</td>
<td>Test2</td>
<td>Gate1</td>
</tr>
<tr>
<td>2</td>
<td>NOT</td>
<td>Gate1</td>
<td></td>
<td>Output1</td>
</tr>
<tr>
<td>3</td>
<td>OR</td>
<td>Test3</td>
<td>Gate1</td>
<td>Output2</td>
</tr>
</tbody>
</table>

Figure 8.12  Sample logic circuit.
Figure 8.13 River and frog for Problem 8-23.
Figure 8.14  Thread pool for Problem 8-24.
Figure 9.1 Finding the rank in parallel.
Figure 9.2 Parallelizing the rank computation.
Parallel Programming: Techniques and Applications using Networked Workstations and Parallel Computers
Barry Wilkinson and Michael Allen © Prentice Hall, 1998

Figure 9.3  Rank sort using a master and slaves.

- Master
- Slaves
- Read numbers
- Place selected number
- a[]
- b[]
Figure 9.4  Compare and exchange on a message-passing system — Version 1.
Figure 9.5  Compare and exchange on a message-passing system — Version 2.
Figure 9.6 Merging two sublists — Version 1.
Figure 9.7  Merging two sublists — Version 2.
Figure 9.8 Steps in bubble sort.
**Figure 9.9** Overlapping bubble sort actions in a pipeline.
<table>
<thead>
<tr>
<th>Step</th>
<th>P₀</th>
<th>P₁</th>
<th>P₂</th>
<th>P₃</th>
<th>P₄</th>
<th>P₅</th>
<th>P₆</th>
<th>P₇</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>8</td>
<td>3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>5</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

**Figure 9.10**  Odd-even transposition sort sorting eight numbers.
Smallest number

Smallest number

Largest number

Figure 9.11 Snakelike sorted list.
Figure 9.12  Shearsort.
Figure 9.13 Using the transpose operation to maintain operations in rows.
Figure 9.14  Mergesort using tree allocation of processes.
Figure 9.15  Quicksort using tree allocation of processes.
Figure 9.16  Quicksort showing pivot withheld in processes.
Figure 9.17 Work pool implementation of quicksort.
(a) Phase 1

(b) Phase 2

(c) Phase 3

Figure 9.18  Hypercube quicksort algorithm when the numbers are originally in node 000.
Figure 9.19  Hypercube quicksort algorithm when numbers are distributed among nodes.
Figure 9.20  Hypercube quicksort communication.
Figure 9.21  Quicksort hypercube algorithm with Gray code ordering.
Odd indices

Even indices

Sorted lists

Merge

Merge

Even indices

Odd indices

Compare and exchange

Final sorted list

Figure 9.22  Odd-even merging of two sorted lists.
Figure 9.23 Odd-even mergesort.
Figure 9.24  Bitonic sequences.
Figure 9.25 Creating two bitonic sequences from one bitonic sequence.
Figure 9.26  Sorting a bitonic sequence.
Figure 9.27  Bitonic mergesort.
Parallel Programming: Techniques and Applications using Networked Workstations and Parallel Computers
Barry Wilkinson and Michael Allen © Prentice Hall, 1998

Figure 9.28  Bitonic mergesort on eight numbers.
Figure 9.29  Compare-and-exchange algorithm for Problem 9-5.

Terminates when insertions at top/bottom of lists

Step 1

Step 2

Step 3

88
50
28
25

98
80
43
42

50
42
28
25

98
88
80
43

43
42
28
25

98
88
80
50
Figure 10.1 An $n \times m$ matrix.
Figure 10.2  Matrix multiplication, $C = A \times B$. 

Row

Column

Multiply

Sum results

$A$  $\times$  $B$  $=  C$

$i$

$j$

$c_{i,j}$
Figure 10.3 Matrix-vector multiplication $c = A \times b$. 
Figure 10.4  Block matrix multiplication.
\[
\begin{bmatrix}
    a_{0,0} & a_{0,1} \\
    a_{1,0} & a_{1,1} \\
    a_{2,0} & a_{2,1} \\
    a_{3,0} & a_{3,1}
\end{bmatrix}
\times
\begin{bmatrix}
    a_{0,2} & a_{0,3} \\
    a_{1,2} & a_{1,3} \\
    a_{2,2} & a_{2,3} \\
    a_{3,2} & a_{3,3}
\end{bmatrix}
\times
\begin{bmatrix}
    b_{0,0} & b_{0,1} \\
    b_{1,0} & b_{1,1} \\
    b_{2,0} & b_{2,1} \\
    b_{3,0} & b_{3,1}
\end{bmatrix}
\]

(a) Matrices

\[
\begin{align*}
A_{0,0} & = \begin{bmatrix}
    a_{0,0} & a_{0,1} \\
    a_{1,0} & a_{1,1}
\end{bmatrix} \\
B_{0,0} & = \begin{bmatrix}
    b_{0,0} & b_{0,1} \\
    b_{1,0} & b_{1,1}
\end{bmatrix} \\
A_{0,1} & = \begin{bmatrix}
    a_{0,2} & a_{0,3} \\
    a_{1,2} & a_{1,3}
\end{bmatrix} \\
B_{1,0} & = \begin{bmatrix}
    b_{2,0} & b_{2,1} \\
    b_{3,0} & b_{3,1}
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
A_{0,0}B_{0,0} & = \begin{bmatrix}
    a_{0,0}b_{0,0} + a_{0,1}b_{1,0} \\
    a_{1,0}b_{0,0} + a_{1,1}b_{1,0}
\end{bmatrix} \\
A_{0,1}B_{1,0} & = \begin{bmatrix}
    a_{0,2}b_{2,0} + a_{0,3}b_{3,0} \\
    a_{1,2}b_{2,0} + a_{1,3}b_{3,0}
\end{bmatrix}
\end{align*}
\]

(b) Multiplying \(A_{0,0} \times B_{0,0}\) to obtain \(C_{0,0}\)

Figure 10.5 Submatrix multiplication.
Figure 10.6  Direct implementation of matrix multiplication.
Figure 10.7  Accumulation using a tree construction.
Figure 10.8  Submatrix multiplication and summation.
Figure 10.9  Movement of $A$ and $B$ elements.
Figure 10.10  Step 2 — Alignment of elements of $A$ and $B$. 
Figure 10.11  Step 4 — One-place shift of elements of $A$ and $B$. 
Figure 10.12 Matrix multiplication using a systolic array.
Figure 10.13 Matrix-vector multiplication using a systolic array.
Figure 10.14  Gaussian elimination.
Figure 10.15  Broadcast in parallel implementation of Gaussian elimination.
Figure 10.16  Pipeline implementation of Gaussian elimination.
Figure 10.17  Strip partitioning.
Figure 10.18  Cyclic partitioning to equalize workload.
Solution space

\[ f(x, y) \]

Figure 10.19 Finite difference method.
Figure 10.20  Mesh of points numbered in natural order.
Those equations with a boundary point on diagonal unnecessary for solution

To include boundary values and some zero entries (see text)

\[ \begin{pmatrix}
1 & 1 & -4 & 1 & 1 \\
1 & 1 & -4 & 1 & 1 \\
1 & 1 & -4 & 1 & 1 \\
1 & 1 & -4 & 1 & 1 \\
1 & 1 & -4 & 1 & 1 \\
\end{pmatrix} \times \begin{pmatrix}
x_1 \\
x_2 \\
x_{N-1} \\
x_N \\
0 \\
0 \\
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix} \]

*Figure 10.21* Sparse matrix for Laplace’s equation.
Figure 10.22  Gauss-Seidel relaxation with natural order, computed sequentially.
Figure 10.23 Red-black ordering.
Figure 10.24 Nine-point stencil.
Figure 10.25  Multigrid processor allocation.
Ambient temperature at edges of board = 20°C

**Figure 10.26** Printed circuit board for Problem 10-18.
Figure 11.1 Pixmap.
Figure 11.2  Image histogram.
<table>
<thead>
<tr>
<th>$x_0$</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_3$</td>
<td>$x_4$</td>
<td>$x_5$</td>
</tr>
<tr>
<td>$x_6$</td>
<td>$x_7$</td>
<td>$x_8$</td>
</tr>
</tbody>
</table>

**Figure 11.3** Pixel values for a $3 \times 3$ group.
Figure 11.4  Four-step data transfer for the computation of mean.
Figure 11.5  Parallel mean data accumulation.
Figure 11.6  Approximate median algorithm requiring six steps.
Figure 11.7 Using a $3 \times 3$ weighted mask.

<table>
<thead>
<tr>
<th>Mask</th>
<th>Pixels</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_0$</td>
<td>$x_0$</td>
<td></td>
</tr>
<tr>
<td>$w_1$</td>
<td>$x_1$</td>
<td></td>
</tr>
<tr>
<td>$w_2$</td>
<td>$x_2$</td>
<td></td>
</tr>
<tr>
<td>$w_3$</td>
<td>$x_3$</td>
<td></td>
</tr>
<tr>
<td>$w_4$</td>
<td>$x_4$</td>
<td></td>
</tr>
<tr>
<td>$w_5$</td>
<td>$x_5$</td>
<td></td>
</tr>
<tr>
<td>$w_6$</td>
<td>$x_6$</td>
<td></td>
</tr>
<tr>
<td>$w_7$</td>
<td>$x_7$</td>
<td></td>
</tr>
<tr>
<td>$w_8$</td>
<td>$x_8$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 11.8  Mask to compute mean.

\[
k = \frac{1}{9}
\]

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]
Figure 11.9 A noise reduction mask.

\[ k = \frac{1}{16} \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 8 & 1 \\ 1 & 1 & 1 \end{array} \]
Figure 11.10  High-pass sharpening filter mask.

\[ k = \frac{1}{9} \]

\[ \begin{array}{ccc}
-1 & -1 & -1 \\
-1 & 8 & -1 \\
-1 & -1 & -1 \\
\end{array} \]
Figure 11.11  Edge detection using differentiation.
Gray level gradient and direction.

Figure 11.12  Gray level gradient and direction.
Figure 11.13  Prewitt operator.
Figure 11.14  Sobel operator.
Figure 11.15  Edge detection with Sobel operator.
<table>
<thead>
<tr>
<th>0</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 11.16** Laplace operator.
Figure 11.17  Pixels used in Laplace operator.
Figure 11.18  Effect of Laplace operator.
Figure 11.19  Mapping a line into \((a, b)\) space.
Figure 11.20  Mapping a line into \((r, \theta)\) space.
Figure 11.21 Normal representation using image coordinate system.
Figure 11.22  Accumulators, acc[\( r \)][\( \theta \)], for the Hough transform.
Figure 11.23  Two-dimensional DFT.
**Figure 11.24** Convolution using Fourier transforms.

(a) Direct convolution

(b) Using Fourier transform
Figure 11.25  Master-slave approach for implementing the DFT directly.
Figure 11.26 One stage of a pipeline implementation of DFT algorithm.
Figure 11.27  Discrete Fourier transform with a pipeline.

Parallel Programming: Techniques and Applications using Networked Workstations and Parallel Computers
Barry Wilkinson and Michael Allen © Prentice Hall, 1998
250
Figure 11.28 Decomposition of \( N \)-point DFT into two \( N/2 \)-point DFTs.
Figure 11.29 Four-point discrete Fourier transform.
Figure 11.30 Sixteen-point DFT decomposition.
Figure 11.31 Sixteen-point FFT computational flow.
Figure 11.32  Mapping processors onto 16-point FFT computation.
Figure 11.33 FFT using transpose algorithm — first two steps.
Figure 11.34  Transposing array for transpose algorithm.
Figure 11.35  FFT using transpose algorithm — last two steps.
Figure 11.36  Image for Problem 11-3.
Figure 12.1  State space tree.
Figure 12.2  Single-point crossover.
Subpopulation

Migration path; every island sends to every other island

Figure 12.3  Island model.
Island subpopulations

Limited migration path

Figure 12.4 Stepping stone model
Figure D.1 PRAM model.
Figure D.2  List ranking by pointer jumping.
Figure D.3 A view of the bulk synchronous parallel model.
Figure D.4 LogP parameters.