

**Joint Optimization of Product Price, Display Orientation and Shelf-Space  
Allocation in Retail Category Management\***

**Chase C. Murray**

Ph.D. Candidate

Department of Industrial & Systems Engineering

State University of New York at Buffalo

308A Bell Hall

Buffalo, NY 14260-2050

(716) 536-2770

cmurray3@buffalo.edu

**Debabrata Talukdar**

Associate Professor

Department of Marketing

State University of New York at Buffalo

215 E Jacobs Management Center

Buffalo, NY 14260-4000

(716) 645-3243

dtalukda@buffalo.edu

**Abhijit Gosavi**

Assistant Professor

Department of Engineering Management & Systems Engineering

Missouri University of Science and Technology

219 Engineering Management

Rolla, MO 65409-0370

(573) 341-4624

gosavia@mst.edu

This Version: February 12, 2010

Second Version: November 9, 2008; First Version: March 30, 2008

---

\* The authors acknowledge the helpful comments and suggestions received when this work was presented at the Production and Operations Management Society (POMS) Annual Conference (May, 2007), and at the Institute for Operations Research and the Management Sciences (INFORMS) Annual Meeting (November, 2007). They are also grateful to the Co-Editors and two anonymous reviewers for their very constructive feedbacks on the earlier versions of this manuscript.

## **Joint Optimization of Product Price, Display Orientation and Shelf-Space**

### **Allocation in Retail Category Management**

#### **ABSTRACT**

We develop a model that jointly optimizes a retailer's decisions for product prices, display facing areas, display orientations and shelf-space locations in a product category. Unlike the existing shelf-space allocation models that typically consider only the width of display shelves, our model considers both the width and height of each shelf, allowing products to be stacked. Furthermore, as demand is influenced by each product's 2-dimensional facing area, we consider multiple product orientations that capture 3-dimensional product packaging characteristics. That enables our model to not only treat shelf locations as decision variables, but also retailers' stacking patterns in terms of product display areas and multiple display orientations. Further, unlike the existing studies which consider a retailer's shelf-space allocation decisions independent of its product pricing decisions, our model allows joint decisions on both and captures cross-product interactions in demand through prices. We show how a branch-and-bound based MINLP algorithm can be used to implement our optimization model in a fast and practical way.

*Keywords:* Shelf-space allocation; Retail category management; Pricing and revenue management; MINLP model.

## INTRODUCTION

In this study, we focus on developing marketing decision support systems for strategic category management by consumer packaged goods (CPG) retailers. The CPG retail industry represents an annual market size of about half a trillion dollars in the USA and is a significant part of the household consumption expenditure (US Census Bureau 2006). Several mutually reinforcing trends in this industry in recent years have made the issue of efficient shelf allocation systems as one of the most critical marketing and operational decisions for the CPG retailers (Chen et al. 1999; Hall, Kopalle and Krishna 2010; Levy et al. 2004; Levy and Weitz 1995).

One such trend is the fact that competition for shelf space in CPG retail stores is at an all time high (Ball 2004). This tremendous demand for shelf space is driven by the competitive need for retailers to introduce new products or categories. Since the 1990s, there has been a significant proliferation in new product items or so called “store keeping units” (SKUs) in supermarkets as both manufacturers and retailers see it as a strategic way for increasing respective market shares (Kurt 1993; Drèze et al. 1994). Retailers have also increasingly ventured into new categories (e.g., organic products) to satisfy consumer needs (Tarnowski 2007). The average number of different items stocked by a CPG supermarket store had increased by 20% between 1970 and 1980, and by 75% between 1980 and 1990 (Greenhouse 2005). These new products and categories are putting a huge demand on the available shelf space which is practically fixed for the existing stores.

At the same time, the CPG supermarket retailers have seen a steady increase in their operational costs from carrying the aforesaid large assortment of SKUs and as a consequence of competition from lower cost, lower assortment-carrying discount retailers (Mullin 2005).

According to a report by the Federal Reserve Bank of Philadelphia, conventional supermarkets accounted for 73% of retail grocery sales in 1980. But by 1994, they accounted for only 28% of the sales after yielding significant market shares to discount “warehouses” and “superstores” (Greenhouse 2005). This increased competitive environment has forced supermarkets to enhance customer services. The hours of operation of an average supermarket has now reached about 19 hours a day. And while retailers have been able to reduce the number of clerks stocking shelves, they had to increase the number of checkout clerks, cash registers and offer more services to their customers which in effect increased their operating costs tremendously.

To control such spiraling costs from increased levels of competition and product assortments in an industry already marked by one of the thinnest net profit margins, CPG retailers are under significant pressure to improve their operational efficiency. In pursuit of their operational efficiency gain, a key reality faced by CPG retailers is the fact that "the majority of consumer decision making occurs in the store" (Drèze et al. 1994). While product sales prices obviously influence such in-store decisions of consumers (Russell and Peterson 2000), past studies have also found that product locations and facing areas can influence consumers' attentions and thereby their purchase decisions (Drèze et al. 1994). For instance, a recent study (Chung et al. 2007) regarding milk sales in New York showed that a 7% increase in milk sales could be realized by effective shelf management techniques, including increasing the visibility of products.

Not surprisingly, it is becoming extremely important for retailers to be as efficient as possible in how they allocate their existing shelf space, possibly the most scarce and strategically valuable of their operational resources (Chen et al. 1999; Turcsik 2003; Urban 1998). The goal has become how best to organize their product assortments to generate more

profit contributions from their existing, limited shelf space (Greenhouse 2005; Lim et al. 2004). A key imperative to achieving such a goal by retailers obviously hinges on their ability to have a good decision support system tool to help with more efficient allocation of their available product shelf space.

The traditional shelf-space management tool employed by retailers is a *planogram*, which provides heuristics for shelf layout of products. However, as Lim et al. (2004) points out, “due to the problem’s complexity, only relatively simple heuristic rules have been developed and are available for retailers to plan product-to-shelf allocation (Zufryden 1986; Yang 2001)...these are not effective global optimization tools (Desmet and Renaudin 1998) and are largely used for planogram accounting to reduce time spent on manual manipulation of shelves (Drèze et al. 1994; Yang 2001).” Borin et al. (1994) note that "substantial" sales could be lost by retailers who rely on such simple heuristic rule-based planograms. Further, as the *Category Management Association* (<http://www.cpgcatnet.org/page/62774/>; accessed 1/29/2010) notes, retailers often rely on product manufacturers to provide planograms. Besides carrying intrinsic bias in favor of the involved manufacturers, these planograms often do not take into account a retailer's specific demand parameters. Thus, retailers need an unbiased, reliable decision support system tool to help them make their shelf-space allocation decisions.

A review of the relevant academic literature (which we discuss in the next section) shows some definite progress in addressing the retailers’ shelf-space allocation problem through studies that have developed formal, systematic approaches. However, such studies continue to be limited in number and they often abstract away from some of the key reality aspects of the allocation problem. Our study’s goal is to extend this existing limited stream of research that systematically addresses the shelf-space allocation problem. It does so by addressing, within a

single study, several key shortcomings in the existing studies when it comes to capturing the critical realistic decision trade-offs faced by CPG retailers in optimizing their shelf space. Specifically, we develop a model that jointly optimizes a retailer's decisions for product prices, display facing areas, display orientations and shelf-space locations in a product category.

Our model contributes to the existing literature in several key ways. Unlike the existing shelf-space allocation models that typically consider only the width of display shelves, our model considers both the width *and* height of each shelf, allowing products to be stacked. The visible facing area allocated to each product is determined by the product's orientation on the shelf, which captures the 3-dimensional product packaging characteristics. That enables our model to not only treat shelf locations as decision variables, but also retailers' stacking patterns in terms of product display areas and multiple display orientations. Further, unlike the existing studies which consider a retailer's shelf-space allocation decisions independent of its product pricing decisions, our model allows joint decisions on both and captures cross-product interactions in demand through prices. Consequently, our model is able to capture the key realistic trade-offs encountered by a retailer among product prices, shelf-locations, display facing areas and display orientations in making its category management decisions.

The remainder of this paper is organized as follows. Section 2 summarizes the relevant existing literature and puts the contribution of our study in its context. Section 3 presents the conceptual framework and the analytical formulation of our joint pricing and shelf-space allocation model. We describe the solution procedure for our proposed model in Section 4. The solution procedure is used to numerically investigate a variety of shelf-allocation problems in Section 5. Section 6 concludes with future research directions.

## LITERATURE REVIEW

Among the existing studies in the retail shelf-space optimization literature, perhaps the most well-known study is that by Corstjens and Doyle (1981). In their model, demand for each product is a function of own- and cross-*space* elasticities (rather than *price* elasticities). Thus, the amount of space allocated to each product determines the demand for all products. The model seeks to maximize the profits of a retailer subject to a capacity limit on total shelf space (which equates to considering a single shelf) and upper and lower bounds on individual product quantities. Corstjens and Doyle (1983) extend their static model to a dynamic one in which product growth potentials are considered to be important factors in allocation decisions. Their dynamic model incorporates exogenously-determined product prices in the demand function.

Bultez and Naert (1988) extend Corstjens and Doyle's static model (1981) by including product-class sales-share elasticities in addition to overall product-class elasticities. Space for each product is given as a percentage of total shelf space, which means integer-valued solutions are not guaranteed by this model and only a single shelf is effectively modeled. Their demand function for each product is an implicit function of space allocation only, although both direct and cross effects are included. Another extension of the Corstjens and Doyle (1981) model is developed by Bookbinder and Zarour (2001). Here, "direct product profitability" is incorporated to capture the profit contributions on an individual SKU level. However, there are no shelf location effects in this model, although own- and cross-space elasticities are included.

The model proposed by Zufryden (1986) includes own-product space effects and demand-related marketing variables but ignores cross-elasticities in demand. To guarantee integer-valued allocation quantities, the shelf is broken into "slots" such that each product's size is a multiple of the slot sizes. This is the only model to address the idea of stackable

products, although it does not allow for multiple display orientations or joint price optimization.

In Borin et al. (1994), demand for each product is characterized by “unmodified”, “modified”, “acquired” and “stock-out” demand. Unmodified demand reflects the consumer’s preference for a product, whereas modified demand equals unmodified demand plus demand generated/lost by space, price, advertising, and promotional impacts. However, they assume that modified demand is solely a function of space allocation, such that own- and cross-price elasticities are ignored. Acquired demand describes sales resulting from products not selected as part of the product assortment. Stock-out demand describes sales resulting from another product being out of stock.

Urban (1998) presents a single-product inventory level optimization problem where demand is a function of inventory displayed, distinguishing between “backroom” inventory and “on-shelf” inventory. Price is considered given and on-shelf inventory is assumed to be replenished instantaneously, provided that backroom inventory exists. This model is extended to a multi-product shelf-space allocation problem. It is assumed that each product can only be allocated to a certain region of the shelf.

Yang and Chen (1999) simplify the Corstjens and Doyle (1981) model by assuming that the demand of each product is linear as long as the number of facings for that product is between some lower and upper bounds. This assumption is inherently problematic as the available inventory and/or minimum required number of facings in reality are likely to be such that the linearity condition is violated. Although there are no cross-effects, this is the first shelf-space model to include location effects. Lim et al. (2004) present two extensions to the Yang and Chen (1999) model. First, they consider product groupings, where cross-product

affinity is modeled as a linear combination of the Yang and Chen (1999) profit function and an additional profit (or cost) term. Second, they consider a general nonlinear profit function.

Another nonlinear model is provided by Bai and Kendall (2005), where demand is a function of the amount of displayed inventory and the space elasticity of the product. However, this model ignores both cross-product demand interaction effects as well as shelf-location effects.

While the above noted existing studies have made significant progress in addressing the shelf-allocation problem, they also fall short in capturing several key aspects of the problem. For one, except for the model by Zufryden (1986), all of the existing shelf-allocation models consider shelves in 1-dimensional (width) space only. As a result, their focus is on the number of product facings while ignoring the stacking process (thus, facing area and multiple display orientations) as retailers' decision variables. However, that significantly abstracts away from the reality of retailers' decision context. For instance, Drèze et al. (1994) clearly find that a product's facing area being displayed, and aesthetic elements of its display (such as size and color coordination related to display orientations of its packaging) are critical decision variables for CPG retailers' shelf space management. Specifically, on the notion that the demand of a product item is driven more by its facing *area* rather than by its *number* of facings, they note: "Number of facings is a good measure intuitively because it is easy to understand and communicate, however it is quite item dependent. One facing for a big item will not have the same effect as one facing for a small item. Holding constant other packaging factors, people are much more likely to visually acquire larger sized targets/products..."

Another limitation of the existing models is their failure to consider a retailer's pricing and shelf-space allocation decisions jointly. That, again, reflects a significant abstraction away from the reality of the decision context for retail category management (Levy and Weitz 1995).

Further, the existing studies model cross-product effects on demand only through relative number of facings instead of relative prices across products. While there is no existing study that directly demonstrates cross-price effects to be more dominant than cross-facing effects on consumers' purchase decision process, indirect empirical evidence appears to support that. Such evidence in the CPG market consists of most consumers' stated propensity to engage in price search across items within a category as well as their revealed propensity reflected through significant cross-price elasticities across items within a category (Bimolt et al. 2005; Bucklin et al. 1998; Mace and Neslin 2004; Urbany et al. 1996). Finally, as evident from the relative paucity of empirical studies (Van Dijk et al. 2004), data for estimating cross-facing elasticities across product items in a category are much less readily available in reality than the sales scanner data needed for estimating cross-price elasticities. Thus, any category management decision support model that captures cross-product demand interactions through selling prices, rather than through display facings, will be much easier to implement in practice.

It should be recalled that even the model by Zufryden (1986), which considers 2-dimensional geometry of shelf and stacking, does not allow for multiple product orientations or joint price optimizations. In summary, the existing studies on retail shelf-space allocation, albeit limited, definitely offer a strong foundation to systematically address this important and interesting business management problem. At the same time, they also abstract away from some of the key decision trade-offs inherent in the reality of this problem faced by CPG retailers. As noted earlier, the goal of our study is to present a systematic approach to the problem that takes into account, within a single framework, those key trade-offs in terms of product prices, display facing area, display orientations and shelf-space locations.

## MODEL DEVELOPMENT

As Drèze et al. (1994) note, the shelf-space allocation problem is quite different depending on whether one takes the perspective of manufacturers or retailers. From the manufacturers' perspective, they want to maximize the sales and profits of their own products, and as such always want more and better shelf space only for their products. In contrast, retailers want to maximize category sales and profits, regardless of manufacturer identities of the products in a category. Our shelf-space allocation model is formulated from the perspective of category management by CPG retailers (Chen et al. 1999). We first discuss its underlying conceptual framework and then its analytical formulation.

### **Conceptual Framework**

As is typical in the existing studies on shelf-space management, we assume that the composition of the product assortment offered by a retailer in a category has already been selected. Specifically, we consider a retailer selling a pre-selected assortment of  $N$  distinct product items in a given product category. The distinctiveness of the product items is based on all the unique combinations of product (brand and attributes) and packaging. Thus, a distinct packaging dimension always implies a distinct product item, but not necessarily vice versa. For example, two distinct product items may still have identical packaging dimensions, but they are distinct product items because they represent different brands or different product attributes (e.g., flavors). The demand for each product item within the category is modeled here as not only a function of its own price but also of the prices of all other items offered in that category. It is modeled also to depend on its display location in the shelf, facing area displayed and display orientations of its packaging. The demand function is modeled to incorporate the insights from the existing relevant empirical research in identifying the role of key strategic

managerial decision variables. Such research not only finds the obvious strong effects of own-price on the demand of a product item, but also that the demands are inter-related across items through cross-price effects (Bimolt et al. 2005; Bucklin 1998).

Further, as noted earlier, the empirical study by Drèze et al. (1994) on shelf-space management in the CPG industry show that several non-price factors influence the demand for a displayed product item. They include the item's shelf-location within a display, its facing area being displayed, and aesthetic elements of its display such as size and color coordination related to display orientations of its packaging. In other words, demand of an item is affected not only by its total display facing area, but also by the "quality" of that area in terms of how it is combined with possible shelf-space locations and display orientations. From a managerial perspective, that implies some form of quality adjusted rank orders for various product display location and orientation combinations. For example, a full facing packaging orientation on an eye-level shelf is likely to be much better in terms of quality of a given amount of display facing area allocated to the product than a side facing packaging orientation on a bottom shelf.

For shelf-display, each product item  $i$  may be rotated about one or more axes to get different orientations for its displayed face to consumers. Let the set  $J_i$  represent all of the allowable orientations for product  $i$  in the given category. Depending on physical contours of typical product packaging, the number of such allowable orientations is likely to vary across categories. For a particular orientation  $j \in J_i$  of one unit of product item  $i$ , the width of a facing is given by  $x_{ij}$ . Similarly, the height (depth) of a facing of product  $i$  in orientation  $j$  is given by  $y_{ij}$  ( $z_{ij}$ ). Therefore, the visible or facing area of one unit of product  $i$  placed in orientation  $j$  is  $x_{ij}y_{ij}$ . Figure 1 shows a product item in three such possible orientations. It is relevant to note here that in our conceptual framework, the product items do not necessarily need to have

physically rectangular contours for their facing areas in any orientation. For a non-rectangular physical contour of a product item in a particular orientation, our model considers the dimensions of the least-area rectangular contour that can “fit” the non-rectangular contour.

[INSERT FIGURE 1 ABOUT HERE]

For a given category, the retailer has a multi-shelf display unit consisting of  $K$  shelves which may be of different dimensions. Unlike the existing shelf-space allocation models which treat a display shelf as 1-dimensional of certain length (e.g., Lim et al. 2004; Yang and Chen 1999; Yang 2001; Bai and Kendall 2005), our model explicitly considers the 2-dimensional available facing area of each shelf and implicitly accounts for the 3-dimensional geometries of products. Specifically, let shelf  $k$  have a width of  $X_k$ , a height of  $Y_k$ , and a depth of  $Z_k$ , giving a total available display area of  $X_k Y_k$ . In this context, it is relevant to recall that the demand for a product item, based on past studies, is independent of the number of units that are placed as on-shelf inventory *behind* the faced products but depends on its display area facing the customers (Drèze et al. 1994). So, in this model we do not consider the number of units that are placed as on-shelf inventory behind the faced products. Instead, we focus solely on the units facing the customers for a product item and assume that good logistics are sufficient to eliminate its out-of-stock occurrences (Yang and Chen 1999; Lim et al. 2004). Our use of product- and shelf-depth dimensions is solely for the purpose of ensuring that each product, when placed in a particular orientation, does not exceed the depth of the shelf.

In contrast to the existing shelf-space allocation models, our model allows us to capture the retailer’s stacking and orientation process for product display as decision variables. The retailer can stack multiple facings of a product item in multiple orientations on a particular shelf. Units of product item  $i$  may only be stacked on top of other units of item  $i$ , and all units

must be in the same orientation in a given stack. If a product is placed on a shelf in a particular orientation, the retailer wishes to stack up as many units of that product in that orientation as physically possible. As such, we define the parameter  $v_{ijk} = \lfloor Y_k / y_{ij} \rfloor$  to represent the integer part of the fraction of the height of shelf  $k$  over the height of product  $i$  when placed in orientation  $j$ . It thus denotes the maximum number of units of product  $i$  that can be placed in orientation  $j$  on shelf  $k$  given the geometries of the product and the shelf. Figure 2 shows an example of a three-shelf display unit in which four different product types are stacked on the shelves in multiple orientations.

[INSERT FIGURE 2 ABOUT HERE]

For the retailer, the strategic goal is to maximize the total profit from a category through joint optimal decisions about the selling prices and “quality adjusted” display facing areas (which includes decisions about shelf locations and display orientations) for the pre-selected assortment of  $N$  product items in the category. The goal is subject to constraints induced by the physical realities of the  $K$  display shelves as well as by the market realities of the decision environment. For example, a market based constraint is likely to be that the selling price for each product item has some lower and upper bounds based on market competition dynamics. We next discuss the analytical formulation of the aforesaid category profit optimization problem faced by the retailer.

### **The Optimization Model**

As noted above, our joint optimization model for the retailer seeks to determine optimal selling prices, shelf locations, facing areas and display orientations for each product item in the category being analyzed. Because our model allows product stacking and multiple product orientations, we need integer decision variables to describe the number of facings of product

item  $i$  in orientation  $j$  that are placed on shelf  $k$ . Let  $f_{ijk}$  be the number of units of product item  $i$  in orientation  $j$  that are placed *directly on* (touching) shelf  $k$ . For example, as shown in Figure 2, there is one unit of product item 1 in orientation 1 that is placed directly on shelf 2 ( $f_{1,1,2} = 1$ ) while five units of the same product item in the same orientation are stacked on top of that unit ( $v_{1,1,2} = 6$ ).

The total number of units of product item  $i$  that are faced (visible) on shelf  $k$  is thus given by  $\sum_{j \in J_i} v_{ijk} f_{ijk}$ , and the total number of facing units of product item  $i$  that are allocated to the display unit (the collection of all shelves) is given by  $\sum_{k=1}^K \sum_{j \in J_i} v_{ijk} f_{ijk}$ . The width of shelf  $k$  that is consumed by product item  $i$  then equals  $\sum_{j \in J_i} x_{ij} f_{ijk}$ . Let  $p_i$  and  $c_i$  denote the retailer's unit selling price and cost respectively for product item  $i$ . As discussed in our earlier conceptual framework, we will assume that demand for product item  $i$  is a function of the selling prices of all product items, its shelf-locations, facing area, and display orientations. Accordingly, let  $d_i(\mathbf{p}, \mathbf{f}_i)$  represent the demand function for product item  $i$ , where  $\mathbf{p} = \{p_1, \dots, p_N\}$  and  $f_{ijk} \in \mathbf{f}_i$  for all  $j \in J_i$  and  $k = 1, \dots, K$ .

We use the following specific non-linear demand function for product item  $i$ :

$$d_i(\mathbf{p}, \mathbf{f}_i) = \alpha_i \left( \sum_{j \in J_i} \sum_{k=1}^K \delta_{ijk} x_{ij} y_{ij} v_{ijk} f_{ijk} \right)^{\beta_i} \prod_{n=1}^N p_n^{\mu_n} \quad (1)$$

where  $\alpha_i > 0$  is a scaling parameter for product item  $i$ . The facing area allocated to product  $i$  in display orientation  $j$  on shelf  $k$  is represented by  $x_{ij} y_{ij} v_{ijk} f_{ijk}$ . The parameter  $\delta_{ijk}$  denotes the shelf location-orientation quality adjustment weight corresponding to the display facing area allocated to product  $i$  in orientation  $j$  on shelf  $k$ , and  $\delta_{ijk} \geq 1$  being normalized with respect to the worst quality weight. The parameter  $\beta_i$  captures the facing area elasticity of product item  $i$ ,

and lies in the domain  $0 < \beta_i < 1$  to capture the diminishing marginal effect of facing area on demand (Drèze et al. 1994). Note that if the facing area elasticity parameter were determined on a product level as well as on a location level, the nature of the resulting polynomial form of the demand function would encourage splitting a product across shelves (Bai and Kendall 2005). For that reason, we use  $\beta_i$  instead of  $\beta_{ik}$ . The own- and cross-price elasticity parameters of demand for product items  $i$  and  $n$  are represented by  $\mu_{in}$  such that  $\mu_{ii} \leq 0$  and  $\mu_{in} \geq 0$  for all  $i \neq n$ . The cross-price elasticity values recognize the fact that product items within a given category will be substitutes rather than complements of each other.

The category profit optimization problem for the retailer is then formulated as:

$$\begin{aligned} \text{Max}_{\mathbf{p}, \mathbf{f}} \quad & \sum_{i=1}^N (p_i - c_i) d_i(\mathbf{p}, \mathbf{f}_i) \end{aligned} \quad (2)$$

$$\text{s.t.} \quad \sum_{i=1}^N \sum_{j \in J_i} x_{ij} f_{ijk} \leq X_k \quad \text{for } k = 1, \dots, K \quad (3)$$

$$f_{ijk} = 0 \quad \text{for all } i, j, k \text{ such that } v_{ijk} = 0 \quad (4)$$

$$f_{ijk} = 0 \quad \text{for all } i, j, k \text{ such that } \lfloor Z_k / z_{ij} \rfloor = 0 \quad (5)$$

$$L_i \leq \sum_{j \in J_i} \sum_{k=1}^K v_{ijk} f_{ijk} \leq U_i \quad \text{for } i = 1, \dots, N \quad (6)$$

$$p_i^{\min} \leq p_i \leq p_i^{\max} \quad \text{for } i = 1, \dots, N \quad (7)$$

$$f_{ijk} \in \{0, 1, 2, \dots\} \quad \text{for } i = 1, \dots, N \quad (8)$$

The decision variables in the objective function (2) are  $p_i$  (continuous) and  $f_{ijk}$  (integer). The various constraints to the optimization problem are captured by (3) - (8). Constraints (3) - (5) are essentially induced by the physical realities in terms of geometries of the display shelves and possible product item orientations. Specifically, constraint (3) states that the width of all product items added to shelf  $k$  must not exceed the width of shelf  $k$ . Constraints (4) and (5) ensure that if a product item's height or depth, when placed in orientation  $j$ , exceeds the height or depth of a shelf, it can not be placed on that shelf in this orientation. In constraint (5),

$\lfloor Z_k / z_{ij} \rfloor$  represents the integer part of the fraction of the depth of shelf  $k$  over the depth of product  $i$  when placed in orientation  $j$ . These constraints could be removed if we simply do not define the  $f_{ijk}$  decision variables for situations where product  $i$  is too large to fit on shelf  $k$  when placed in orientation  $j$ .

Constraints (6) and (8) reflect the key likely market realities in the retailer's decision environment. Specifically, constraint (6) states that the number of facings of product  $i$  must be between some lower and upper bounds. As noted in the literature (e.g., Yang 2001), the lower bound  $L_i$  captures the retailer's contractual obligations with product manufacturers in the category that require that a minimum amount of facing area is assigned to each product item in the category assortment. Because the retailer has already pre-selected the product mix (i.e.,  $L_i > 0$ ), this model is not suitable for product selection. The upper bound,  $U_i$ , may be used to capture a retailer's desire to phase out a particular product. Based on relevant competitive market dynamics, constraint (7) imposes lower and upper bounds on the allowable selling prices for each product item  $i$ , where  $p_i^{\min}$  ( $p_i^{\max}$ ) is the lower (upper) bound. Finally, constraint (8) ensures that the number of facings must be integer valued, consistent with the reality of the decision context.

## SOLUTION METHODOLOGY

A variety of solution approaches have been proposed for the shelf-space allocation models reviewed earlier in Section 2. Some of these approaches do not result in integer-valued allocation quantities, thus making them unsuitable for implementation by a retailer. For example, Corstjens and Doyle (1981) use a geometric programming technique via a heuristic (Gochet and Smeers, 1979). Bookbinder and Zarour (2001) also apply geometric

programming, as well as Kuhn-Tucker multipliers. Finally, Bultez and Naert (1988) use marginal analysis and a search heuristic.

An optimization approach resulting in integer-valued allocation quantities can be found in Zufryden (1986), where dynamic programming is used to allocate products to pre-defined “slots” in the shelving area. Yang (2001) proposes a greedy knapsack heuristic, augmented by three adjustment phases. Although this was the first optimization approach to include location effects, it relied on a linear objective function. Lim et al. (2004) extend Yang’s (2001) single-product adjustment heuristic by employing multiple-product neighborhood moves. They also use a network flow solution, tabu search, and a so-called “squeaky wheel” optimization procedure (Joslin and Clements 1999) to handle nonlinear objective functions.

Other optimization approaches for mixed integer nonlinear objective functions include genetic algorithms and simulated annealing. For example, Urban (1998) uses a greedy search heuristic and a genetic algorithm, and Borin et al. (1994) use simulated annealing to solve their shelf-space allocation problems. Bai and Kendall (2005) also make use of simulated annealing, but it is used to select among twelve low-level heuristics that either add, delete, swap, or interchange products.

Our mixed non-linear model (see Equation (1)) features a non-convex, non-separable objective function with linear constraints (see (2)-(8)). In general, mixed integer nonlinear problems (MINLPs) are difficult to solve, and unless the problem is convex, there is no known method that can guarantee optimality (Bussieck and Pruessner 2003). Dynamic programming (DP) approaches can also be used to solve integer programs and their variants. They are easy to implement when the objective function is separable in decision variables and the solution space is relatively small. However, if the function is non-separable and the solution space is

large, DP suffers from the curse of dimensionality, which renders it ineffective. Since our objective function is non-separable and the feasible solution space is huge, the curse of dimensionality applies, and DP is ruled out as a viable solution technique. Another approach that could be potentially adopted here is a meta-heuristic one (e.g., Bai and Kendall 2005, 2009). While the use of a meta-heuristic approach is beyond the scope of our current study, an interesting future research direction would be to explore such alternative solution techniques.

The structure of our problem makes it very amenable to solutions with heuristics that have been developed for mixed integer non-linear programming problems. BONMIN (Basic Open Source Nonlinear Mixed Integer programming) is a collection of computer programs based on algorithms, some of which are heuristic, that have been written to solve problems which have the mixed-integer, nonlinear flavor (Bonami and Lee 2007). It is available from the COIN-OR (Computational INfrastructure) libraries (<http://www.coin-or.org>). BONMIN relies on several mathematically sound nonlinear and integer programming concepts. For prior use of BONMIN, see e.g., Braggali et al. (2006) for design of water distribution networks, Almadi et al. (2008) for hyperplane clustering of data points, and Bonami and Lejeune (2007) for financial portfolio optimization. We note that in the absence of convexity, BONMIN algorithms do not guarantee convergence to optimality, and hence they serve as heuristics for our problem. However, they include computational features that significantly improve the quality of solutions even for non-convex problems (Bonami and Lee 2007).

BONMIN provides three specific algorithms that are of interest to us in solving MINLPs that we encounter in the context of our proposed model. The first one, “B-BB,” is a basic branch-and-bound algorithm in which a continuous nonlinear programming relaxation is solved at each node of the search tree. The second so-called “B-OA” algorithm features an

outer-approximation based decomposition in which the objective function and constraints are linearized at various points, and a mixed integer *linear* programming relaxation is obtained. The third algorithm, “B-Hyb”, is a hybrid of the outer-approximation and branch-and-cut algorithms. Technical details on all BONMIN options can be found in Bonami et al. (2005).

We should note that the scale of our optimization problem depends on the values of three variables: number of product items ( $N$ ) in the category; possible types of product display orientations ( $J$ ); and number of shelves ( $K$ ) in the display unit. For our numerical analysis, we chose “large-scale” problem cases that include very high values for all the three variables consistent with the reality of CPG retailers’ decision environment. For instance, existing studies (e.g., Russell and Petersen 2000, p.387) note that a typical CPG retailer in the USA carries about 600 product categories and the average number of distinct SKUs carried in a category is around 50. Our direct information from a large regional CPG supermarket chain in the Northeast USA also suggests that that is indeed the case with the number of SKUs in a category rarely exceeding 100. Accordingly, we allow  $N$  to have a value as high as 100. Also, based on field observations in CPG supermarkets, we choose the maximum values of  $J$  and  $K$  to be 3 and 10 respectively.

We should also note that the maximum number of decision variables for any problem that we study is given by  $N + \sum_{i=1}^N |J_i|K$ , where  $|J_i|$  represents the cardinality of the set  $J_i$ .

There are exactly  $N$  continuous decision variables for prices ( $p_i$ ), and at most  $\sum_{i=1}^N |J_i|K$  integer decision variables for product facings ( $f_{ijk}$ ). For example, the *maximum* number of decision variables for a 3-product, 3-orientation, 2-shelf problem is  $3 + 3 \times 3 \times 2 = 21$ . However, the *actual* number of decision variables required for this problem could be lower if shelf  $k$  is not

sufficiently large to accommodate product  $i$  in orientation  $j$ , since in that case the corresponding  $f_{ijk}$  decision variable is not needed to be defined.

## NUMERICAL ANALYSIS

For our computational experimental set-up, we generated test problems with a wide range of parameter values. Our approach to generating the random problem instances is similar to that adopted by Yang (2001) and Lim et al. (2004). Our goal here is to study the relative performance of the three relevant BONMIN algorithms noted earlier with respect to their performance in terms of the objective function values and computational time. We perform our computational experiments in two stages. In the first stage, we perform experiments with small-scale problems, which have relatively small solution spaces, to see how all three of these algorithms perform. The advantage of running the experiments on the small-scale cases first is that one can compare the quality of the solution with respect to a bound on the objective function value. If the performance of any of these three algorithms turns out to be unsatisfactory in the small-scale cases, we can exclude it from our experiments for the large-scale problems, which form the second stage. All numerical testing was performed via BONMIN version 0.99.2 on a PC running Ubuntu Linux 7.10 with 1.83 GHz Intel Core Duo processors and 2 GB RAM.

Table 1 shows some of the values for system parameters used for the small-scale and the large-scale problems. Where indicated in Table 1, parameter values were determined according to a uniform distribution. For each product  $i$ ,  $\beta_i$  was uniformly distributed such that  $0 < \beta_i < 1$ , while  $c_i$  was uniformly distributed between  $0.8p_i^{\min}$  and  $p_i^{\min}$ . To maintain consistency with the bounds on facing quantities ( $L_i$  and  $U_i$ ) the values of the  $\alpha_i$  (scaling) parameters were calculated as  $\alpha_i = U_i / \prod_{n=1}^N \bar{p}_i^{\mu_{in}} (\bar{\delta}_i x_{i1} y_{i1} (L_i + U_i) / 2)^{\beta_i}$ , where

$\bar{p} = (p_i^{\min} + p_i^{\max})/2$  and  $\bar{\delta}_i = \sum_{j \in J_i} \sum_{k=1}^K \delta_{ijk} / (|J_i|K)$ . The dimensions of the shelves were calculated such that sufficient space was available to allow a feasible solution while still imposing binding limitations on the number of allocated facings.

[INSERT TABLE 1 ABOUT HERE]

For the price elasticity parameters, we impose the following usual assumption in the context of multi-product inter-related demand functions in a category (Talluri and van Ryzin, p.

324):  $|\mu_{ii}| \geq \sum_{\substack{n=1 \\ n \neq i}}^N |\mu_{ni}|$ . This assumption ensures that we do not allow for the counter-intuitive

market outcome case where an increase in the overall category demand can be generated by in fact raising the price of a product in the category. It is important to note that this assumption does allow for the expected market outcomes of an increase in overall category demand when product prices decrease and vice versa. For our numerical analysis, the own-price elasticity parameters,  $\mu_{ii}$ , were uniformly distributed between -1.5 and -1 for each product  $i$ . Consistent with the aforesaid assumption, the associated cross-price elasticity parameters,  $\mu_{in}$ , were uniformly distributed between 0 and  $-\mu_{ii}/(N-1)$  for  $n \neq i$ .

To implement our proposed decision support model in practice, it is of course imperative that a retailer uses reliable estimates of the model parameters that recognize the empirical realities of the specific demand-side characteristics faced by the retailer. In this respect, as in other decision support models for retailers (e.g., Silva-Risso, Bucklin and Morrison 1999), all the parameters in our model are quite amenable to empirical estimations based on typical natural and/or experimental field data available to retailers. For instance, the price parameters,  $\mu$ , can easily be estimated from the routine scanner data of actual purchase transactions available to retailers (Mace and Neslin 2004; Russell and Peterson 2000).

As for the shelf location-orientation quality parameters,  $\delta$ , and the facing area elasticity parameters,  $\beta$ , they are conceptual counterparts to the usual space elasticity parameters in the existing shelf-space allocation models (Lim et al. 2004). The reason our model has the aforesaid two distinct parameters, as opposed to the usual single space elasticity parameter in the existing allocation models, is because our model entails not only shelf-space allocation decisions but display orientation decisions as well. At the same time, both the parameters,  $\delta$  and  $\beta$ , could be estimated by retailers based on similar natural and/or experimental field data used to estimate shelf-space elasticity parameters (Drèze et al. 1994; Frank and Massy 1970). The techniques will essentially involve some regression based estimation of product demands with relevant covariates. Such covariates will include continuous measures like product facing area per se as well as categorical measures capturing various combinations of product display orientations and shelf-space locations. In a broader analogy, the empirical estimation techniques will be similar to the ones used to evaluate the effectiveness of advertising in terms of alternative contents, media, duration, etc. (Tellis et al. 2005).

It is pertinent to point out that although the parameters in our demand function were defined to be as general as possible in order to capture a wide range of decision environments faced by a retailer, it is likely that the dimensionality of some of these parameters will be smaller in most real-life situations. This will significantly reduce the parameter space and its data requirements. For instance, although the parameter  $\delta_{ijk}$  allows for different quality-adjustment weight values for each product item, display orientation and shelf location combinations (e.g., children's cereal might be better placed on a low shelf, while adult cereals would be better at a higher eye level), such values can be independent of product items in most realistic cases. In such cases, we can replace  $\delta_{ijk}$  by  $\delta_{jk}$ .

Results for small-scale and large-scale problems are shown in Table 2 and Table 3, respectively. For the small-scale problems, we used four values for the 3-tuples,  $(N, J_i, K)$ , which are enumerated in the first column of Table 2. For each problem size  $(N, J_i, K)$  shown in each row in Table 2, we generate 32 different sample problems by varying the parameters described in Table 1. Thus, we have a total of 128 sample problems for the small cases. Similarly, we generate 128 sample problems for the large cases. The specific four values of the 3-tuples,  $(N, J_i, K)$ , for the large scale problems are enumerated in the first column of Table 3.

[INSERT TABLE 2 AND TABLE 3 ABOUT HERE]

The relative performance results of the three relevant BONMIN algorithms on the 128 test problems of various sizes are described in Table 2. For the B-BB algorithm, we configured BONMIN to generate 500 random starting solutions. The B-Hyb and B-OA algorithms also work better with a “good” starting solution, which we generated via a heuristic described in the Appendix. A time limit of 50 minutes was imposed on the B-BB algorithm, a reasonable limit on the time that a retailer would have for solving a problem of the dimension considered in the small cases. The B-BB algorithm reached the 50-minute time limit without finding an optimal solution on 63 out of the 128 test problems. Hence we compared the objective function value of each method to a BONMIN-generated bound on the value of the objective function; the bound is generated via a non-linear relaxation. The “Performance Ratio” for each algorithm for a given problem size  $(N, J_i, K)$  is computed as the average over all 32 problem instances of the following quantity: the objective function value generated by that algorithm divided by the bound. The last row of Table 2 shows that when we increase the size of the problem and use  $(10, 3, 5)$  for  $(N, J_i, K)$ , the B-Hyb and B-OA algorithms actually start outperforming the B-BB algorithm. Further, Table 2 also shows that the B-Hyb and B-OA algorithms do not need more

than 37 seconds – a small fraction of the allotted 50 minutes of run time. Hence, it is quite clear that for large-scale problems, which a retailer will likely face in the real world, the B-BB algorithm does not appear to be a viable approach. Therefore, we use the B-Hyb and B-OA algorithms in our experiments with the large-scale problems.

Table 3 shows the average and maximum amount of time (in seconds) required by the B-Hyb and B-OA algorithms for the various large scale test problems. The B-Hyb and B-OA algorithms generated identical solutions to all of the test problems. Only slight differences in runtimes were observed. Our results indicate that these two algorithms provide solutions in a very reasonable amount of time, even for large-scale problems. The average solution times are especially attractive when one notes that the typical decision cycle for retailers in this context will be weekly.

As our product demand function models the interactive influences of and thus trade-offs among multiple decision variables, viz., prices (both own and cross), allocation quantity, orientation, and location, it does not lend itself to generating “generic” rules for retailers’ shelf-space allocation problem. It also underscores the impracticality of traditional “rules of thumb” for shelf-space allocations like using a ranking of products on the basis of profit margin per unit area (Borin et al. (1994) as product margins and display areas need to be decision variables rather than *a priori* parameters. At the same time, an analysis of our numerical results reveals some common characteristics of optimal shelf-space assignments that are in line with general intuition and economic rationale. First, products that are allocated near their upper allowable bound ( $U_i$ ) tend to be placed in orientations with the largest facing area, and in the most attractive shelf locations for that specific product-orientation combination. Second, products with lower profit margins,  $(p_i - c_i)$ , tend to be placed in orientations with reduced facing areas.

Consistent with economic theory and intuition, we also observe that the optimal prices are set by our model in such a way that the realized demand approaches the allocation quantity for each product. For instance, given the overall shelf-space limitations which are further compounded by lower bounds on the allocation quantities for low-demand products, demands for highly-desirable products are likely to exceed their possible allocation quantities. Based on the trade-off between a product's profit contribution and allotted shelf-space that is reflected in our model structure, the optimal prices for such high-demand products are found to be set at or near their upper limits ( $p_i^{max}$ ). In other words, as expected from economic intuitions of relevant trade-offs embedded in our model structure, we observe that optimal prices for high-demand products tend towards their upper bounds, while optimal prices for low-demand products tend towards their lower bounds. With appropriate price bounds, our model can thus effectively identify the category profit maximizing price premiums and discounts for the products offered.

## CONCLUSION

A confluence of recent market trends has made it all the more important for CPG retailers to be as efficient as possible in managing the allocation of their existing shelf space, arguably the most scarce and strategically valuable of their operational resources. The goal has become how best to organize their product assortments to generate more profit contributions from their existing, limited shelf space. A key imperative to achieving such a goal by retailers obviously hinges on their ability to have a good decision support system tool to help with more efficient allocation of their available product shelf space. In this study, we focused on developing such a decision support system tool.

We develop a model that jointly optimizes a retailer's decisions for product prices, display facing areas, display orientations and shelf-space locations in a product category. As a

result, our model better captures the realistic decision environment faced by retailers, including several key aspects hitherto ignored by the existing models. First, the existing shelf-space allocation models only consider the width of display shelves and do not allow for product stacking and multiple display orientations. In contrast, our model considers both the width *and* height of the display shelves, where the quantity of units that may be stacked depends upon each product's chosen display orientation. That enables our model to not only treat shelf locations as decision variables, but also retailers' stacking patterns in terms of product display areas and multiple display orientations. Further, unlike the existing studies which consider a retailer's shelf-space allocation decisions independent of its product pricing decisions, our model allows joint decisions on both and captures cross-product interactions in demand through prices. We show how a branch-and-bound based MINLP algorithm can be used to implement our optimization model in quite a fast and practical way.

As ours is the first known joint optimization model for the pricing and allocation aspects of the shelf-space problem, several interesting opportunities exist for future research. For one, as noted earlier, an interesting future research direction would be to explore alternative solution techniques like the meta-heuristic approaches (Bai and Kendall 2005, 2009) and compare them with our proposed approach in terms of solution efficiency. Also, in terms of the substantive structure of our model, it could be extended to allow mixed-orientation stacking in the same vertical column for products of the same type. Additionally, future models could include decision contexts that call for determining the optimal number and height of adjustable shelves. In other words, future model structures could allow the "geometries" of the display shelves to be decision variables as well.

While our model focuses on a retailer with a pre-selected product assortment, another interesting but challenging future research possibility would be to modify the model to perform product selection decisions simultaneously with display decisions. This would require significant changes to the proposed demand function to avoid the case where a product is not allocated to the shelf but its price still influences the demand for other products. Finally, in a broader context, our proposed shelf-space allocation problem could be integrated with upstream supply chain logistics, thus capturing the inventory management aspects of retailers' category management problem.

## REFERENCES

- Amaldi, Edoardo, Alberto Ceselli, Kanika Dhyani (2008), "Column Generation for the Minimum Hyperplane Clustering Problem," Proceedings of the 7<sup>th</sup> Cologne-Twente Workshop on Graphs and Combinatorial Optimization. Gargano, Italy, May 13-15, 2008. pp 162-171.
- Bai, Ruibin and Graham Kendall (2005), "An Investigation of Automated Planograms Using a Simulated Annealing Based Hyper-heuristics," Pp 87-108 in *Metaheuristics: Progress as Real Problem Solvers - (Operations Research/Computer Science Interfaces, Vol. 32)*. Toshihide Ibaraki, Koji Nonobe, and Mutsunori Yagiura (eds). Berlin, Heidelberg, New York: Springer.
- Bai, Ruibin and Graham Kendall (2009), "A Model for Fresh Produce Shelf Space Allocation and Inventory Management with Freshness Condition Dependent Demand," *INFORMS Journal on Computing*, 20(1):78-85.
- Ball, Deborah (2004), "Consumer-Goods Firms Duel for Shelf Space," *Wall Street Journal*, October 22, 2004.
- Bimolt, Tammo H. A., Harald J. Van Heerde and Reek G.M. Pieters (2005), "New Empirical Generalizations on the Determinants of Price Elasticity," *Journal of Marketing Research*, Vol.38 (May); 141-156.
- Bonami, Pierre and Jon Lee (2007), "BONMIN Users' Manual," (accessed November 4, 2008), [available at <http://www.coin-or.org/Bonmin>].
- Bonami, Pierre, Lorenz T. Biegler, Andrew R. Conn, et al. (2005), "An Algorithmic Framework for Convex Mixed Integer Nonlinear Programs," *IBM Research Report RC23771*, October 31, 2005.

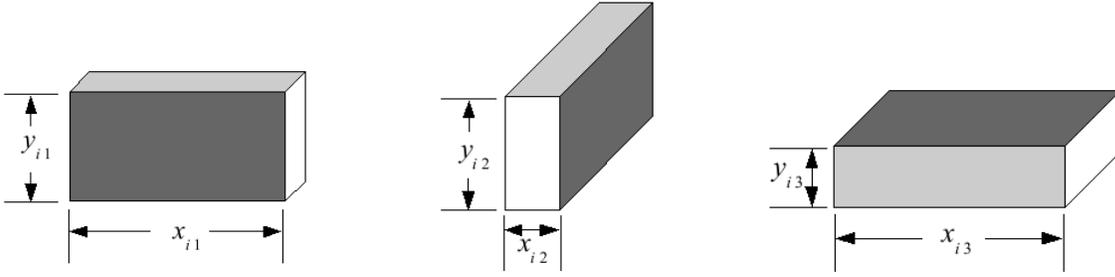
- Bonami, Pierre and M.A. Lejeune (2007), "An Exact Solution Approach for Portfolio Optimization under Stochastic and Integer Constraints," *Operations Research* (In press).
- Bookbinder, James H. and Feyrouz H. Zarour (2001), "Direct product profitability and retail shelf-space allocation models," *Journal of Business Logistics*, 22(2):183-208.
- Borin, Norm, Paul W. Farris, and James R. Freeland (1994), "A model for determining retail product category assortment and shelf space allocation," *Decision Sciences*, 25(3): 359-384.
- Bragalli, Cristiana, Claudia D'Ambrosio, Jon Lee, Andrea Lodi, and Paolo Toth (2006), "An MINLP Solution Method for a Water Network Problem", Algorithms-ESA 2006. Y. Aizer and T. Erlebach, Editors. Proceedings of the 14<sup>th</sup> Annual European Symposium, Zurich, Switzerland, Springer, New York, 2006, 696-707.
- Bucklin, Randolph E., Gary J. Russell, and V. Srinivasan (1998), "A Relationship Between Market Share Elasticities and Brand Switching Probabilities," *Journal of Marketing Research*, 35 (February), 99-113.
- Bultez, Alain and Philippe Naert (1988). "SHARP: Shelf allocation for retailers' profit," *Marketing Science* 7(3), 211-231.
- Bussieck, Michael R. and Armin Pruessner (2003), "Mixed Integer Non-Linear Programming," *SIAG/OPT Views and News*, 14(3),19-22.
- Chen, Yuxin, James D. Hess, Ronald T. Wilcox, and Z. John Zhang (1999), "Accounting versus Marketing Profits: A Relevant Metric for Category Management," *Marketing Science*, 18, 208-229.

- Chung, Chanjin, Todd M. Schmit, Diansheng Dong, Harry M. Kaiser (2007), "Economic Evaluation of Shelf-Space Management in Grocery Stores," *Agribusiness*, 23 (4), 583–597.
- Corstjens, Marcel and Peter Doyle (1981), "A model for optimizing retail space allocations," *Management Science*, 27(7), 822-833.
- Corstjens, Marcel and Peter Doyle (1983), "A dynamic model for strategically allocating retail space," *Journal of the Operational Research Society*, 34(10), 943-951.
- Desmet, Pierre and Valérie Renaudin (1998), "Estimation of Product Category Sales Responsiveness to Allocated Shelf Space," *International Journal of Marketing Research*, 15, 443-457.
- Drèze, Xavier, Stephen J. Hoch, and Mary E. Purk (1994), "Shelf management and space elasticity," *Journal of Retailing*, 70 (4), 301-326.
- Frank, Ronald E and William E. Massy (1970), "Shelf Position and Space Effects on Sales," *Journal of Marketing Research*, 7 (1), 59-66.
- Gochet, Willy and Yves Smeers (1979), "Reversed Geometric Programming: A Branch-and-Bound Method Involving Linear Sub-problems," *Operations Research*, 27 (5): 982-996.
- Greenhouse, Steven (2005), "How Costco Became the Anti-Wal-Mart," *New York Times*, July 17, 2005.
- Hall, Joseph M., Praveen K. Kopalle, and Aradhna Krishna (2010), "Retailer Dynamic Pricing and Ordering Decisions: Category Management versus Brand-by-Brand Approaches," forthcoming, *Journal of Retailing*.

- Joslin, David. E. and David P. Clements (1999), "Squeaky wheel optimization," *Journal of Artificial Intelligence Research*, 10: 353-373.
- Kurt Salmon Associates (1993), "Efficient Consumer Response: Enhancing Consumer Value in the Grocery Industry," *Food Marketing Institute*, Report #9 (Washington DC), 526.
- Levy, Michael, Dhruv Grewal, Praveen K. Kopalle, and James D. Hess (2004), "Emerging Trends in Retail Pricing Practice: Implications for Research," *Journal of Retailing*, 80 (3), xiii-xxi.
- Levy, Michael and Barton A. Weitz (1995), "*Retailing Management*," Chicago: Irwin.
- Lim, Andrew, Brian Rodrigues, and Xingwen Zhang (2004), "Metaheuristics with Local Search Techniques for Retail Shelf-Space Optimization," *Management Science*, 50 (1): 117-131.
- Mace, Sandrine and Scott A. Neslin (2004), "The determinants of pre- and post-promotion dips in sales of frequently purchased goods," *Journal of Marketing Research*, 41 (August), 339-350.
- Mullin, Tracy (2005). "Reinventing Supermarkets," *Chain Store Age*, 81 (7), 28.
- Russell, Gary J. and Ann Peterson (2000), "Analysis of Cross-Category Dependence in Market Basket Selection," *Journal of Retailing*, 76 (3), 367-92.
- Silva-Risso, Jorge M., Randolph E. Bucklin and Donald G. Morrison (1999), "A Decision Support System for Planning Manufacturers' Sales Promotion Calendars," *Marketing Science*, 18 (3), 274-300.
- Talluri, Kalyan.T. and Garrett J. van Ryzin (2005). *The Theory and Practice of Revenue Management*. New York: Springer.
- Tarnowski, Joseph (2007), "Growing, Naturally," *Progressive Grocer*, 86 (13): 70-71.

- Tellis, Gerard J., Rajesh K. Chandy, Deborah J. MacInnis and Pattana Thaivanich (2005), "Modeling the Microeffects of Television Advertising: Which Ad Works, When, Where, for How Long, and Why?," *Marketing Science*, 24 (3), 359-66.
- Turcsik, Richard (2003), "Got Milk, Need Space," *Progressive Grocer*, 82 (1): 49-51.
- Urban, Timothy L. (1998), "An Inventory-Theoretic Approach to Product Assortment and Shelf Space Allocation," *Journal of Retailing*, 74 (1): 15-35.
- Urbany, Joel E., Peter E. Dickson and Rosemary Kalapurakal (1996), "Price Search in the Retail Grocery Market," *Journal of Marketing*, 60 (April), 91-104.
- US Census Bureau (2006), "Estimates of Monthly Retail and Food Services Sales by Kind of Business: 2006," (accessed December 10, 2007), [available at <http://www.census.gov/mrts/www/data/html/nsal06.html>]
- Van Dijk, Albert , Harald J. van Heerde, Peter S.H. Leeflang and Dick R. Wittink (2004), "Similarity-Based Spatial Methods to Estimate Shelf Space Elasticities," *Quantitative Marketing and Economics*, 2 (3): 257-277.
- Yang, Ming-Hsien (2001), "An Efficient Algorithm to Allocate Shelf Space," *European Journal of Operational Research*, 131, 107-111.
- Yang, Ming-Hsien and Wen-Cher Chen (1999), "A study on shelf space allocation and management," *International Journal of Production Economics*, 60-61: 309-317.
- Zufryden, Fred S. (1986), "A Dynamic Programming Approach for Product Selection and Supermarket Shelf Space Allocation," *The Journal of the Operational Research Society*, 37 (4): 413-422.

**FIGURES**

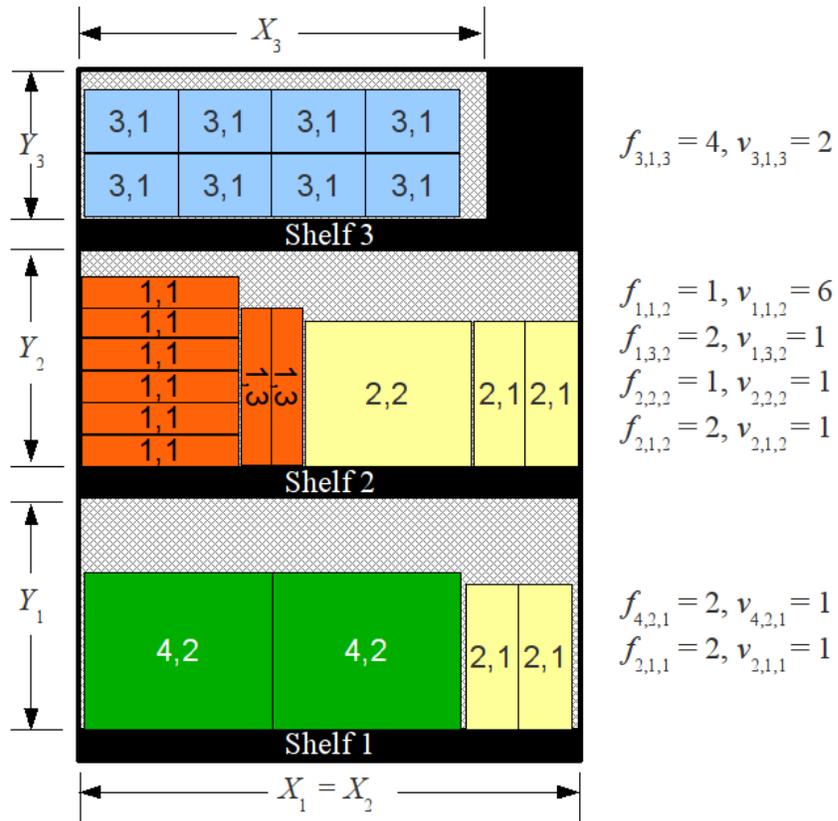


(a) Orientation  $j = 1$ .

(b) Orientation  $j = 2$ .

(c) Orientation  $j = 3$ .

**Figure 1:** Possible orientations for product  $i$ . Note that  $x_{i1} = x_{i3}$ ,  $y_{i1} = y_{i2}$ , and  $x_{i2} = y_{i3}$ .



**Figure 2:** Example of a three-shelf display unit with four product types. The  $i, j$  labels on each product represent the product type ( $i$ ) and orientation ( $j$ ).

## TABLES

**Table 1:** Parameter ranges for creating the numerical examples

Parameter	Value	Range
$(N, J, K)$	$(3,3,2); (5,3,3); (7,3,4); (10,3,5); (25,3,7); (50,3,10); (75,3,10); (100,3,10)$	
$\delta_{ijk}$	Unif(1,D)	D = 2,10
$x_{ij}$	Unif(4,S)	S = 8, 20
$y_{ij}$	Unif(4,S)	S = 8, 20
$L_i$	Unif(a,b)	$(a,b,c,d) = (1,10,10,50)$ ,
$U_i$	Unif(c,d)	$(20,40,50,200)$
$p_i^{\min}$	Unif(P,Q)	$(P,Q,R) = (.5,1,1.5), (5,10,15)$
$p_i^{\max}$	Unif(Q,R)	
$X_k$	$\sum_{i=1}^N L_i \frac{L}{K}$	$L = 4, 10$
$Y_k$	Unif(min_area,max_area)/(KX <sub>k</sub> )	$\min\_area = \sum_{i=1}^N L_i x_{i1} y_{i1}$ , $\max\_area = \sum_{i=1}^N U_i x_{i1} y_{i1}$

**Table 2:** Performance of alternative BONMIN algorithms

$(N, J, K)$	Decision Variables (avg, max)	Time in Seconds (avg, max)			Performance Ratio <sup>a</sup>		
		B-BB	B-Hyb	B-OA	B-BB	B-Hyb	B-OA
(3,3,2)	20.4, 21	121.3, 896	0.4, 2	0.3, 3	0.9852	0.9852	0.9852
(5,3,3)	47.4, 50	1431.0, 3000 <sup>b</sup>	15.4, 33	15.4, 33	0.9883	0.9877	0.9877
(7,3,4)	88.8, 91	2894.6, 3000 <sup>c</sup>	29.3, 35	29.9, 37	0.9884	0.9852	0.9852
(10,3,5)	148.6, 160	3000, 3000 <sup>d</sup>	33.6, 37	33.3, 35	0.9820	0.9827	0.9826

<sup>a</sup> Denotes average of the ratios of objective function values obtained by each method to that obtained by the nonlinear programming relaxation.

<sup>b</sup> 1 of 30 problems reached 3000-second limit.

<sup>c</sup> 30 of 32 problems reached 3000-second limit.

<sup>d</sup> 32 of 32 problems reached 3000-second limit.

**Table 3:** BONMIN solution times for large-scale problems

$(N, J, K)$	Decision Variables (avg, max)	Time in Seconds (avg, max)	
		B-Hyb	B-OA
(25,3,7)	538.8, 550	68.8, 610	69.8, 617
(50,3,10)	1491.5, 1550	192.3, 1036	192.8, 1007
(75,3,10)	2216.8, 2325	664.4, 3091	662.9, 3086
(100,3,10)	2997.8, 3100	993.1, 2645	991.8, 2716

## APPENDIX

### A greedy heuristic to generate starting solutions for the B-Hyb and B-OA algorithms.

*Step 1 – Initialization.*

$$p_i = (p_i^{\min} + p_i^{\max})/2 \quad \forall i = 1, \dots, N.$$

$$f_{ijk} = 0 \quad \forall i = 1, \dots, N, j \in J_i, k = 1, \dots, K.$$

Remaining horizontal shelf space  $S_k = X_k \quad \forall k = 1, \dots, K.$

Number of allocated facings,  $F_i = 0 \quad \forall i = 1, \dots, N.$

Calculate rough estimate of each product's profit per width:

$$P_{ijk} = (p_i - c_i) \frac{\alpha_i}{x_{ij}} (\delta_{ijk} x_{ij} y_{ij} v_{ijk})^{\beta_i} \prod_{n=1}^N p_n^{\mu_n} \quad \forall i = 1, \dots, N, j \in J_i, k = 1, \dots, K.$$

*Step 2 – Allocate each product up to its lower bound on the number of facings.*

Until ( $F_i \geq L_i$  for  $i = 1, \dots, N$ ) {

For  $i = 1, \dots, N$  {

$$j, k = \arg \max_{j, k} \{P_{ijk} : x_{ij} \leq S_k, y_{ij} \leq Y_k\}$$

If ( $j, k = \emptyset$ ) {  $F_i = U_i$  (Product  $i$  will not fit on any shelf.) }

Else {

$$f_{ijk} = f_{ijk} + 1 \quad (\text{Add column of product } i \text{ in orientation } j \text{ to shelf } k).$$

$$F_i = F_i + v_{ijk} \quad (\text{Update the allocation quantity of product } i).$$

$$S_k = S_k - x_{ij} \quad (\text{Reduce remaining available shelf space}).$$

}

} }

*Step 3 – Allocate each product up to its upper bound on the number of facings.*

Repeat Step 2, replacing the outermost comparison of " $F_i \geq L_i$ " by " $F_i = U_i$ ".