CASE STUDY FOR VENDOR-MANAGED INVENTORY

(BASED ON SUI, GOSAVI, & LIN, 2010)

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As discussed in class, in Vendor-Managed Inventory (VMI) systems, the vendor is responsible for inventory management at the retailer, specifically for ensuring that the retailer does not run out of stock. The retailer shares the point-of-sales (POS) data with the vendor for all the products in the contract. VMI contracts of this nature are often called *consignment inventory VMI contracts*. We will restrict ourselves to only a consignment VMI contract in this case study. This case study is based on the journal article: *Sui, Gosavi, and Lin (2010).*

In this case study, we assume that the distribution center (DC) is owned or controlled by the vendor, and trucks must be sent on regular time intervals to the different retailers (customers) in order to meet demand at the retailer. The key problem is to estimate the demand at each retailer accurately, and then use a rule based in the newsvendor to determine the quantity to be dispatched. Typically, the truck (or fleet of trucks) serve multiple retailers (customers) in each trip, and hence the volume sent via the truck in one trip should serve all the customers visited in that trip.

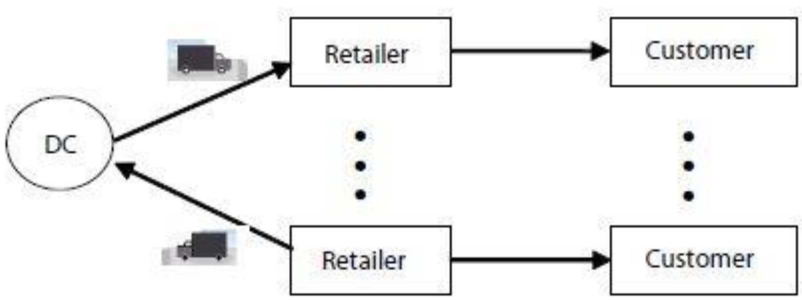

Fig 1. This figure shows the structure of the system studied

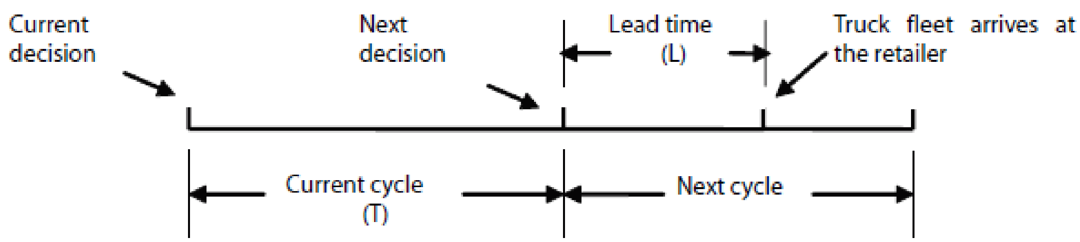

Fig 2. This figure shows how different events unfold in this discrete-event system, as the truck leaves the DC and returns to it, which is regarded as one cycle.

(Both figures from *Sui, Gosavi, and Lin, 2010*)

Model Details:

We first present key notation.

- d: Size (or volume) of demand (from customer) at the retailer
- t: Time between successive arrivals of demand (or customers) at the retailer
- D: Total demand in the retailer in unit time (e.g., one day)
- N: The number of demand arrivals in unit time (e.g., one day) at the retailer
- T: The time taken to complete one cycle by the vendor, serving all the customers in question

In this case study, the following assumptions are made:

- The time between successive arrivals of demand is exponentially distributed.
- The size of the demand, *d*, is uniformly distributed between *a* and *b*.

Then the total customer demand in unit time (e.g., one day) in the retailer is defined by:

$$D = \sum_{i=1}^{N} d_i$$

where d_i denotes the volume of demand from the *i*th arrival during unit time and *N* denotes the number of customers that arrive during unit time; *d* has a uniform distribution with a minimum of *a* and a maximum of *b*. Since the time between successive arrivals is exponentially distributed, *D* becomes what is called a compound Poisson process (Ross, 2002). Let λ denote the Poisson rate of arrival of customers at the retailer in question. Then, the mean μ and variance σ^2 of the customer demand in unit time, *D*, can be given as via the Wald's equation (Ross, 2002):

$$\mu = E(N)E(d) = \frac{\lambda\,(a+b)}{2}$$

$$\sigma^2 = E(N)Var\,(d) + Var(N)E^2(d) = \lambda\,\frac{(b-a)^2}{12} + \lambda\,(\frac{a+b}{2})^2$$

In the above, note that for the Poisson distribution, the mean and the variance are both λ. If the ordering cycle time duration, *T,* is a discrete random variable with mean μ_C and variance σ_C^2, then the mean and variance of the demand during the ordering cycle, D_T, is given as (Nahmias, 2001):

$$\mu_{cycle} = \mu\,.\,\mu_C$$

$$\sigma_{cycle}^2 = \mu_C.\sigma^2 + \mu^2.\sigma_C^2$$

If *T* is large enough, the central limit theorem (Ross, 2002) applies, and then the distribution of D_T can be shown to be normal with a mean of μ_{cycle} and variance of σ_{cycle}^2. When the distribution of the demand is known, one can use the classical newsvendor to determine the optimal order-up-to level, *S*. In the *classical cost-optimal* newsvendor, the value of the optimal order size is determined in the **newsvendor model** (Askin and Goldberg, 2002; Nahmias, 2001) by solving:

$$F(S) = \frac{p}{p+h}$$

where S is the ordering size (or order-up-to level), p is the penalty cost, h is the holding cost, and $F(.)$ is the cumulative distribution function of the customer demand in an ordering cycle. When the distribution of the demand during the ordering cycle is known, one can solve for S to determine the minimum cost solution.

Worked out example with data from the paper (Sui, Gosavi, and Lin, 2010):

T: 30 time units with a probability of 0.25, 40 time units with a probability of 0.5, and 50 time units with a probability of 0.25*

d: UNIF(1,2)

λ: 0.25 per unit time

h: \$0.06 per unit inventory

p: \$4.00

*A distribution of this kind is often used by retailers when exact data are unavailable for trip durations

Then, using the model above,

$$\mu = \frac{\lambda\,(a+b)}{2} = 0.25\ (3)/2 = \ 0.375.$$

$$\sigma^2 = \ \lambda\ \frac{(b-a)^2}{12} + \ \lambda\ (\frac{a+b}{2})^2 \ = 0.25\ \left[\frac{1}{12} + \frac{9}{4}\right] = 0.5833.$$

Then, $\mu_C = (30)(0.25) + (40)(0.5) + (50)(0.25) = 7.5 + 20 + 12.5 = 40.$

And $\sigma_C^2 = \ (30-40)^2(0.25) + (40-40)^2(0.25) + \ \ (50-40)^2(0.25) = 50.$

The above implies that the demand at the retailer during the cycle (time it takes for the truck to complete one trip) has a mean of

$$\mu_{cycle} = \mu \, . \mu_C = (0.375)(40) = 15$$

and a variance of

$$\sigma^2_{cycle} = \mu_C . \sigma^2 + \mu^2 . \sigma^2_C = (40)(0.5833) + 0.375^2(50) = 23.33 + 7.03 = 30.36.$$

- Clearly, if no optimality rule is used, the retailer could send the mean demand, i.e., $S = 15$. This is the so-called MDH (mean demand heuristic).
- If the vendor decides to use six-sigma theory (since we do have the normal distribution here), $S = mean + 3 \, std \, deviations = 15 + 3\sqrt{30.36} = 31.53.$
- If one were to use the newsvendor rule (Askin and Goldberg, 2002), then: $F(S) = \frac{p}{p+h} = \frac{4}{4+0.06} = 0.9852.$ Then, $S = mean + k \, std. deviations,$ where $k = \phi^{-1}(0.98) = 2.17;$ then $S = 15 + 2.17\sqrt{30.36} = 26.96.$

The term project is to work this out for other cases in the dataset provided in *Sui, Gosavi, and Lin (2010).*

REFERENCES

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Nahmias, Steven (2001). *Production and Operations Analysis* (4th ed.). New York: McGraw-Hill (2001).

Ross, Sheldon M (2002). *Introduction to Probability Models* (8 ed.). San Diego, CA: Academic Press.

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