Problem Set 1

1.1. The charge density for a hydrogenic p state can be written as

\[ \rho(r, \theta) = \frac{2}{3} \frac{e}{a_0^3} \frac{1}{64\pi} \left( \frac{r}{a_0} \right)^2 \exp\left( -\frac{r}{a_0} \right) [P_0(\cos \theta) - P_2(\cos \theta)] \]

\[ e = 4.8 \times 10^{-10} \text{ statCoulomb}, \quad a_0 = 0.529 \times 10^{-8} \text{ cm}. \]

(a) Determine the multipole moments of this charge distribution and give the potential for large \( r \) in terms of these moments. (b) Determine the potential, near the origin, correct to order \( r^2 \). (c) Determine the interaction energy with a nuclear quadrupole moment, \( Q = 10^{-24} \text{ cm}^2 \), located at the origin.

1.2: Obtain the transverse and longitudinal components of the current densities:

\[ \mathbf{j}(r, t) = \text{Re} \left\{ e^{-i\omega t} i_0 \left[ \Theta \left( z + \frac{a}{2} \right) - \Theta \left( z - \frac{a}{2} \right) \right] \delta(x) \delta(y) \right\} \mathbf{k} \]

\[ b) \quad \mathbf{j}(r, t) = \text{Re} \left\{ e^{-i\omega t} i_0 \delta(z) \delta(\rho - b) \right\} \Phi, \text{ cylindrical coordinates} \]

(c) For each current density obtain the related charge density (charge conservation) and the potential which vanishes at points infinitely far from the charges.

(d) Verify that Eq. 1.42 is correct.