Problem Set 4:

(Assignment 5) For a given system the electric field and current density are given by Assignment 5

\[ \mathbf{E}(\mathbf{r}, t) = E_0 \exp \left[ -\frac{(z - ct)^2}{2L^2} \right] \cos(k_0(z - ct)) \mathbf{e}_1 \]

\[ \mathbf{J}(\mathbf{r}, t) = J_0 \exp \left[ -\frac{(z - ct)^2}{2L^2} \right] \cos(k_0(z - ct) + \phi_0) \mathbf{e}_1 \]

(a) Calculate the magnetic flux density, \( \mathbf{B}(\mathbf{r}, t) \).

(b) Calculate the frequency dependent fields and current densities \( \mathbf{e}(\mathbf{r}, \omega) \), \( \mathbf{b}(\mathbf{r}, \omega) \), and \( \mathbf{j}(\mathbf{r}, \omega) \).

(c) What are the units of these three quantities in the gaussian units used by Jackson, in the SI (MKSC) units?

(d) Starting with the results of part (b) evaluate \( \mathbf{e}(\mathbf{r}) \), \( \mathbf{b}(\mathbf{r}) \), and \( \mathbf{j}(\mathbf{r}) \) defined in the notes.

(e) Let \( \phi_0 = \pi/6 \) and \( k_0L = 2 \times 10^{12} \pi \) and calculate the time average of \( \mathbf{E}(\mathbf{r}, t) \cdot \mathbf{J}(\mathbf{r}, t) \) over the period from \( t = -T/2 \) to \( t = +T/2 \) at \( z = \pm L \) and at \( z = 0 \). (Note, a tabulated integral)

(f) Plot the results of part (e) for \( 0 < cT/L < 1 \).

(Assignment 6A) Jackson Problem 7.19 -

An approximately monochromatic plane wave packet in one dimension has the instantaneous form, \( u(x, 0) = f(x) \exp(i k_0 x) \) with \( f(x) \) the modulation envelope.

\[ (1) f(x) = N_1 \exp \left( -\frac{a|x|}{2} \right) \]

\[ (2) f(x) = N_2 \exp \left( -\frac{a^2 x^2}{4} \right) \]

\[ (3) f(x) = N_3 (1 - a|x|) \Theta(1 - a^2 x^2) \]

\[ (4) f(x) = N_4 \Theta(1 - a^2 x^2) \]

(a) Calculate the wave number spectrum \( |A(k)|^2 \) for each of the above forms for \( f(x) \).

(b) Explicitly evaluate the rms deviations from the mean, \( \Delta x \) and \( \Delta k \).

(c) Using graphs compare the functions \( |f(x)|^2 \) and the functions \( |A(k)|^2 \).