Problem Set 7.

(Adapted from Jackson 9.10) The transitional charge and current densities for the radiative transition from the $m = 0, 2p$ state in hydrogen to the 1$s$ ground state are (with the neglect of spin),

\[ \rho(r, \theta, \varphi, t) = \frac{2e}{\sqrt{6} a_o^4} r \exp\left(-\frac{3r}{2a_o}\right) Y_{00} Y_{10} \exp(-i\omega_o t); \]
\[ J(r, \theta, \varphi, t) = -\frac{i\nu_o}{2} \left[ \hat{\mathbf{r}} + \frac{a_o}{z^2} \hat{\mathbf{z}} \right] \rho(r, \theta, \varphi, t). \]

where $a_o = 4\pi \epsilon_o h^2/me^2 = 0.529 \times 10^{-10}m$ is the Bohr radius, $\omega_o = 3e^2/32\pi \epsilon_o \hbar a_o$ is the frequency difference of the levels, and $\nu_o = e^2/4\pi \epsilon_o \hbar = \alpha c \approx c/137$ is the Bohr orbit speed.

a) Find the general expression for the vector potential, $\mathbf{A}(r, t)$, and the $\phi(r, t)$ for this localized radiation source.

b) Evaluate all the nonvanishing radiation multipole contributions in the long wavelength limit.

c) Find the magnetic induction, $\mathbf{B}(r, t)$, and the electric field, $\mathbf{E}(r, t)$, in the radiation limit.