Syllabus of Mathematical Physics I (Math 402)
(University of Missouri-Rolla)

(This course is based on a set of notes developed to give the students mathematical "tools" for problem solving. Several references are suggested, but no text is required. All students receive copies of the notes and (after grading) solutions to problems and exams.)

I. Vector Spaces
Definitions of field, vector space, inner product, norm, metric; unitary vector space, normed vector space, orthogonality, Schmidt orthogonalization procedure, linear independence, completeness; basis vectors, dimension; linear transformations, powers of operators; Hermitian conjugate, Hermitian orthogonal and unitary operators; three dimensional Euclidian space; transformations between Cartesian coordinate systems: Euler angle rotations.

II. Generalized Linear Coordinate Transformations
Contravariant and covariant vectors and the transformation matrices, $A_{ij}$; Kronecker delta, $\delta_{ij}$, Levi-Civita symbol, $\varepsilon_{ijk}$; metric tensor, $ds^2=g_{ij}dq^idq^j$; $g=A^T A$, spherical coordinate system, use of $g_{ij}$ to relate covariant and contravariant components, $u^i$ basis vectors in the general $q^i$ coordinates system; obtaining contravariant (covariant) components of $\mathbf{F}$; $\mathbf{F}$, $\mathbf{F}'$ invariance; determinant in summation notation; covariant forms for gradient, Laplacian, curl and divergence; use of covariance in working problems.

III. Tensors Under Generalized Coordinate Transformations
Definition of tensor; rank, symmetric tensors, contraction, quotient rule; tensors with zero components, tensor equations, metric tensors and their determinants; pseudo tensors; transformation of $\varepsilon_{ijk}/\sqrt{g}$.

IV. Vector Analysis
Differential, $d\mathbf{F} = [\mathbf{d} \mathbf{r} \cdot \nabla] \mathbf{F}(\mathbf{r})$; directional derivative, $d^n\mathbf{F}/ds^n = [\mathbf{s} \cdot \nabla]^n \mathbf{F}(\mathbf{r})$; Taylor series expansion, $\mathbf{F}(\mathbf{r} + \delta \mathbf{r}) = \exp[\delta \mathbf{r} \cdot \nabla] \mathbf{F}(\mathbf{r})$; physical interpretation of divergence and curl; manipulation of $\mathbf{r}$, $\mathbf{d} \mathbf{r}$, $\nabla$ in vector equations; vector identities and summation notation; angular momentum operator, $\mathbf{L}$; $\mathbf{L} \times \mathbf{L} = i \hbar \mathbf{L}$; line integrals; surface integrals; $d\mathbf{A} = dx_i dx_j / [(n_i \mathbf{\nabla} f / |\mathbf{\nabla} f|)]$; three dimensional integrals; divergence theorem; Stokes theorem; $\mathbf{\nabla} \times \mathbf{F} = 0 \rightarrow \mathbf{F} = -\mathbf{\nabla} \Phi$; Helmholtz theorem ($\mathbf{F}(\mathbf{r})$ from curl and divergence of $\mathbf{F}$), Maxwell's equations, Dirac delta function, charge densities for simple geometries; step function and its use.

V. Differential Equations and Special Functions
Differential operators, common partial differential equations of physics; techniques for solving partial differential equations; general solution; homogeneous and non-homogeneous equations; Poisson's eq.; solutions to $\nabla \Phi = 0$ in Cartesian, spherical and cylindrical coordinates; plane and spherical waves; Laplacian in terms of angular momentum operator; second order linear ordinary differential equations; Greens' functions in one dimension; boundary conditions; self adjoint operators; hermitian operators; series solution method; Fuch's theorem, general solutions to: hypergeometric, confluent hypergeometric, associated Legendre, Legendre, associated Laguerre, Hermite, Bessel, and spherical Bessel equations; Hankel functions, gamma function; spherical harmonics; expansions of $1/|\mathbf{r} - \mathbf{r}'|$, Schrodinger equation for H atom.
REFERENCES TEXTS FOR MATH 402/403


7. **Mathews, Jon and Walker, R.L.** "Mathematical Methods of Physics" W.A. Benjamin, Inc., N.Y., 1970 This book treats a broad range of advanced topics: ordinary differential equations; infinite series, evaluation of integrals, integral transforms; complex variables, vectors and matrices, special functions; partial differential equations; eigenfunctions, eigenvalues, Green's function perturbation theory, integral equations; calculus of variations probability and statistics, tensor analysis; group theory. This book covers these subjects with very few details. It was meant as an outline for students to follow in pursuing these topics on their own.

8. **Morse, Philip M. and Feshbach, Herman** "Methods of Theoretical Physics" Mc-Graw Hill, N.Y., 1953, Volumes I and II Good reference book; some material for course will be taken from these books - but not enough to warrant buying either volume.

9. **Korn, Granino A. and Korn, Theresa M.** "Mathematical Handbook for Scientists and Engineers" Contains an excellent, concise, and detailed summary of just about any mathematical topic you are likely to encounter in classwork or research. This is a very good reference book! Consider buying this book for your library (not required or advised for this course specifically).


12. **Bergmann, Peter Gabriel** "Introduction to the Theory of Relativity" Dover, New York 1976. Chapter V has a very good (and complete) discussion of Vector and Tensor Calculus....you should find this useful.


14. **Lovelock, David and Rund, Hanno** "Tensors, Differential Forms and Variational Principles".

15. **Cushing, J.T.** "Applied Analytical Mathematics for Physical Scientists" John Wiley and Sons, Inc., New York, 1975. This book is more theoretically than practically oriented -- but it should provide background for the class notes and some extra explanations.

16. **Spain, B. and Smith, M.G.** "Functions of Mathematical Physics" Van Nostrand Reinhold Company, New York, 1970. This is a good reference for Chapter V of class notes: Differential Eqs. This book takes a little different approach to special functions, but you will find it complements material in notes.

17. **Andrews, Larry C.** "Special Function for Engineers and Applied Mathematicians" Macmillan, New York, 1985. Has nice discussion of special functions including the Digamma function, Beta function, fractional derivatives, Gegenbauer polynomials, Jacobi polynomials, incomplete gamma function, Packhammer symbol,........