Class Today

• Print notes and examples
• Theorems of Pappus & Guldinus
• Example Problems
• Group Work Time
Theorems of Pappus and Guldinus

Two theorems describing a simple way to calculate volumes (solids) and surface areas (shells) of revolution are jointly attributed to:

**Pappus of Alexandria**, third century Greek Philosopher

**Paul Guldin or Habakkuk Guldin** (1577-1643)

![Torus Image](http://upload.wikimedia.org/wikipedia/commons/c/c6/Simple_Torus.svg)

By GYassineMrabetTalk; This vector image was created with Inkscape. (Own work)

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The area of a surface of revolution (a shell) equals the length of the generating curve ($L$) times the distance traveled by the centroid of the curve ($D$) while the surface is being generated.

$$A_s = L \cdot D$$
First Theorem of Pappus & Guldinus

• The distance the centroid travels \( (D) \) for a surface area of revolution will be in the shape of a partial or full circle.

• Arc length, \( S \), for a circle is defined as \( S = r \Theta \) where

\[
r = \text{the radius of the circle} \\
\Theta = \text{the angle of the arc}
\]
First Theorem of Pappus & Guldinus

• The distance the centroid travels for a surface area of revolution will be in the shape of a circle.
• Arc length for a circle is defined as $S = r \theta$

$$A_S = L \cdot D$$
$$A_S = \bar{r} \Theta L$$

or for composite shapes:

$$A_S = \Theta \sum (r_i L_i)$$
First Theorem of Pappus & Guldinus

Calculates a surface area ($A_s$), or a shell, of revolution

$$A_s = \Theta \Sigma (\bar{r}_i \, L_i)$$
The volume of a body of revolution (a solid) equals the generating area times the distance traveled by the centroid of the area while the body is being generated.

\[ V = A \cdot D \]
Second Theorem of Pappus & Guldinus

- Once again, the distance the centroid travels is the shape of a circle.

- Substituting ...

\[ S = r \Theta \]

\[ V = A \cdot D \]

\[ V = \vec{r} \Theta A \]

or for composite shapes:

\[ V = \Theta \Sigma (\vec{r}_i A_i) \]
Second Theorem of Pappus & Guldinus

Calculates a volume \((V)\), or a solid, of revolution

\[
V = \Theta \sum (\vec{r}_i A_i)
\]
Terms Defined for Each Equation

\[ A_s = \Theta \sum (\bar{r}_i L_i) \]
\[ V = \Theta \sum \bar{r}_i A_i \]

Where

\( \Theta \) = angle of revolution measured in radians
\( \bar{r}_i \) = distance from axis of revolution to centroid of the generating curve or generating area
\( L_i \) = length of generating curve
\( A_i \) = generating area