Applications of PT Symmetry and Pseudo-Hermiticity: From the Imaginary Cubic Perturbation to Elementary Particle Physics

PHHQP-12 Conference 2013
Koc University

[Thanks to the Organizers
for Recognizing the Need to Have an Event
on Fundamental Physics!]

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In exploring nature,
a possible starting point is the
beauty of mathematics!

British physicist: He/she will do the right approximations, identify the essential degrees of freedom and arrive at a concise formula to describe the phenomenon.

German physicist: He/she will dig to the bottom of things using an apparatus that he/she crafted and wielded together with his/her own hands.

French physicist: He/she will start from mathematics, then explore the beauty of mathematics, and if, in the end, there is an application to be found somewhere, it is purely accidental but welcome.

I never thought that I would ever work on superluminal physics
[superluminal: faster than light]
However, then I made some obervations:

[1.] The superluminal Dirac Hamiltonian,
whose eigenstates satisfy the dispersion
$E^2 = p^2 - m^2$, is pseudo-Hermitian (a PHHQP topic).

[2.] One can solve the superluminal Dirac
equation in the helicity basis. The results have
a conspicuously simple and compact mathematical structure.

[3.] There exist *sum rules* fulfilled by the solutions [2.],
which allow for a calculation of the
field-theoretical propagator of the superluminal Dirac field.

[4.] The results [1.]-[3.] are independent of the OPERA
experiment and hold for infinitesimally superluminal
neutrinos. They are obtained within the "French" approach to physics.

PT Symmetry and Generalized Bender-Wu Formulas

PRL **102,** 011601 (2009) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2009

Unified Treatment of Even and Odd Anharmonic Oscillators of Arbitrary Degree

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We present a unified treatment, including higher-order corrections, of anharmonic oscillators of arbitrary even and odd degree. Our approach is based on a dispersion relation which takes advantage of the $\mathcal{PT}$ symmetry of odd potentials for imaginary coupling parameter, and of generalized quantization conditions which take into account instanton contributions. We find a number of explicit new results, including the general behavior of large-order perturbation theory for arbitrary levels of odd anharmonic oscillators, and subleading corrections to the decay width of excited states for odd potentials, which are numerically significant.

DOI: 10.1103/PhysRevLett.102.011601 PACS numbers: 11.10.Jj, 11.15.Bt, 11.25.Db

$$h_M(g) = -\frac{1}{2}\partial_q^2 + \frac{1}{2}q^2 + \sqrt{g}\,q^M. \qquad (M\ \text{odd}).$$

Negative g: PT Symmetric

The $\mathcal{PT}$ symmetry for purely imaginary coupling leads to the following dispersion relation

$$\epsilon_n^{(M)}(g) = n + \frac{1}{2} + \frac{g}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}\,\epsilon_n^{(M)}(s+i0)}{s(s-g)}.$$

Dirac Hamiltonians

Hermitian Hamiltonian, Subluminal

$$H^{(t)} = \vec{\alpha} \cdot \vec{p} + \beta\, m_1 + \mathrm{i}\, \beta\, \gamma^5\, m_2 \,.$$

$$E = \sqrt{\vec{p}^{\,2} + m_1^2 + m_2^2}.$$

$$H^{(t)} = \left(H^{(t)}\right)^+ ,$$

$$H^{(t)} = \text{Hermitian operator}$$

γ^5 Hermitian Hamiltonian, Pseudo-Hermitian, Superluminal

$$H' = \vec{\alpha} \cdot \vec{p} + \mathrm{i}\, \beta\, m_1 + \beta\, \gamma^5\, m_2 \,,$$

$$E = \sqrt{\vec{p}^{\,2} - m_1^2 - m_2^2}.$$

$$H' = \gamma^5 H'^{+} \gamma^5 .$$

[U.D.J. and B.J.Wundt, arXiv:1205.0521,
ISRN High-Energy Physics 2013 (2013) 374612]

Tachyonic and Tardyonic Dispersion Relations

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **84**, 021806(R) (2011)

$\mathcal{PT}$-symmetry in honeycomb photonic lattices

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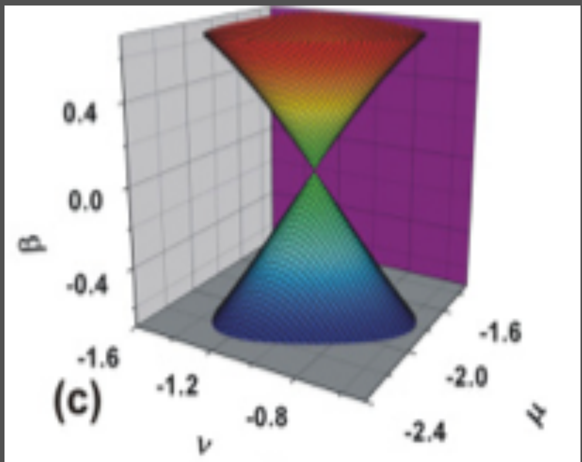
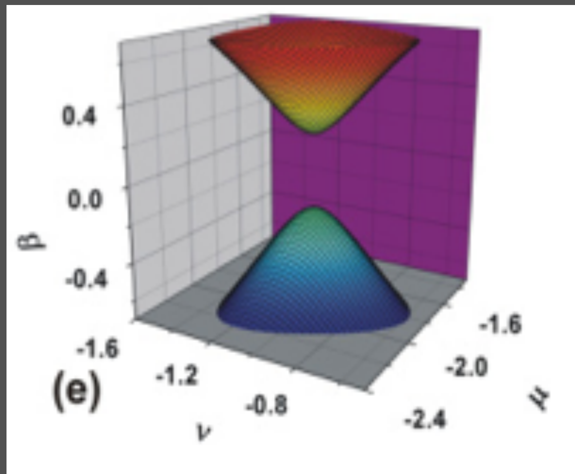
We apply gain and loss to honeycomb photonic lattices and show that the dispersion relation is identical to tachyons—particles with imaginary mass that travel faster than the speed of light. This is accompanied by $\mathcal{PT}$-symmetry breaking in this structure. We further show that the $\mathcal{PT}$-symmetry can be restored by deforming the lattice.

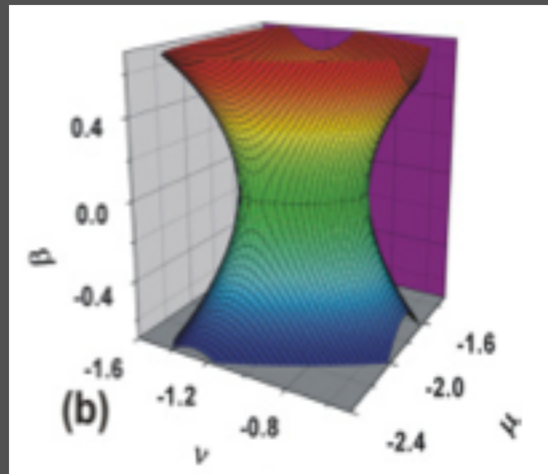
DOI: 10.1103/PhysRevA.84.021806 PACS number(s): 42.25.−p, 42.82.Et



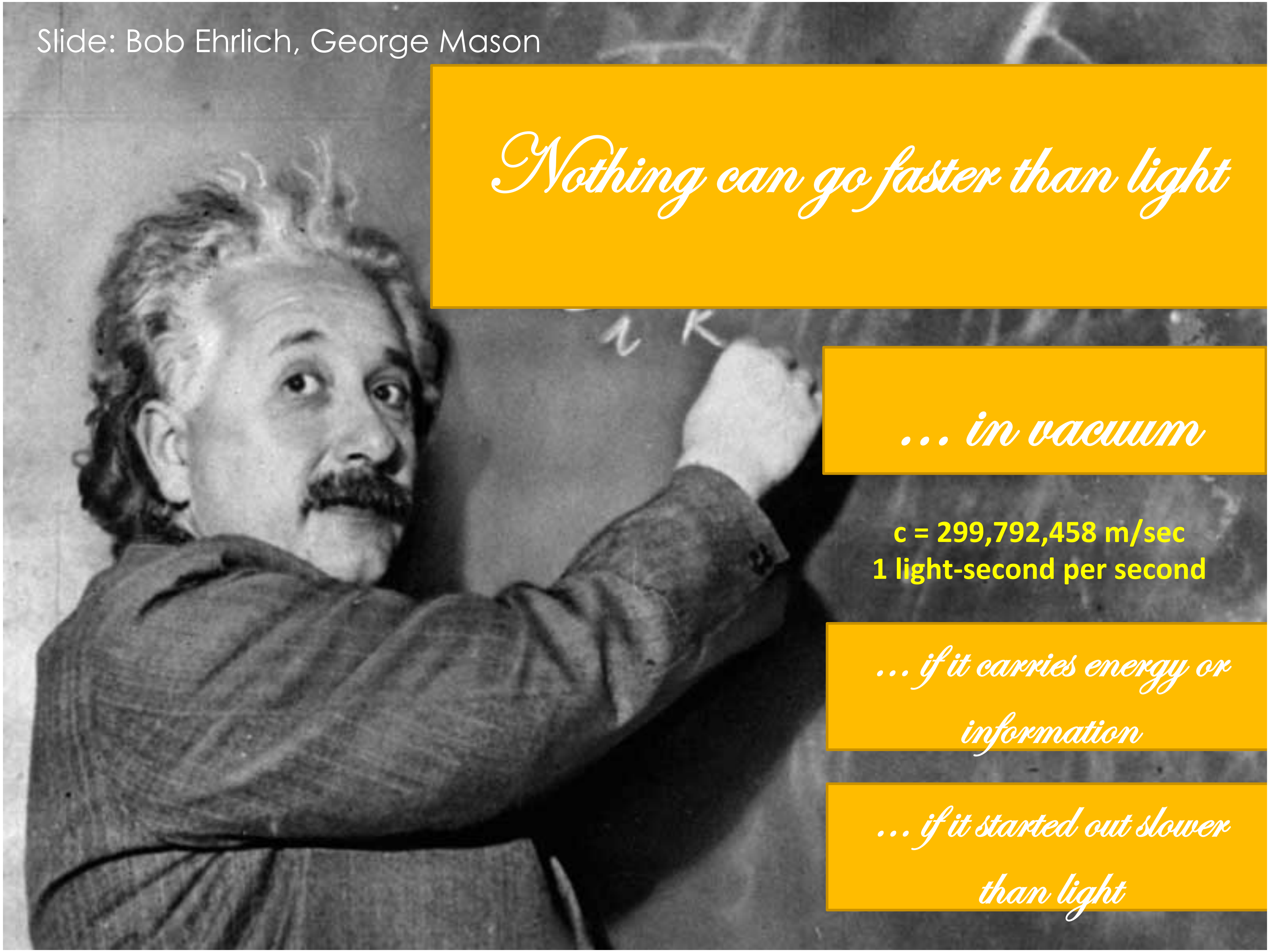
Massless



Tardyonic



Tachyonic


Slide: Bob Ehrlich, George Mason
Nothing can go faster than light
… in vacuum
c = 299,792,458 m/sec
1 light-second per second
… if it carries energy or information
… if it started out slower than light

“Autobahn Paradox” (Known to J.A.Wheeler Already)…

What happens if Dr. Spock overtakes a left-handed neutrino…

“Autobahn Paradox”: A left-handed, massive neutrino (traveling slower than light) appears right-handed once you pass it on a highway.

Wheeler said that the neutrino has to be massless,
beyond doubt, because only strictly massless spin-1/2 particles find a convenient representation in the helicity basis (Weyl fermions).

Wheeler even discouraged experimentalists who intended to measure the neutrino mass. The original standard model (SM) predicted massless neutrinos; the observation of neutrino oscillations implies SM -> SM++.

Four possible ways to resolve the "autobahn paradox":

[1.] The neutrino is strictly massless (Weyl fermion).
But: We have neutrino oscillations.

[2.] The neutrino is its own antiparticle (Majorana fermion).

[3.] The neutrino is an ever so slightly superluminal Dirac particle.

[4.] Exotic mechanisms (sterile neutrinos).

Answer [2.] (currently the preferred answer) would imply that the neutrino behaves differently from any other spin-1/2 particle in the Standard Model. A Majorana neutrino would force us to abandon the concept of lepton number: Muon and antimuon (negative and positive charge) are their respective antiparticles and decay into (essentially) muon neutrino and muon antineutrino. The decay products, however, would count as one and the same, identical particle. The Majorana wave functions would have to fulfill the charge conjugation invariance condition (be "real rather than complex").

Majorana wave functions cannot simply be of the form exp(-i k.x).

Question (Answer [3.]):

Are (at least some of the flavor eigenstates
of the) neutrinos superluminal?

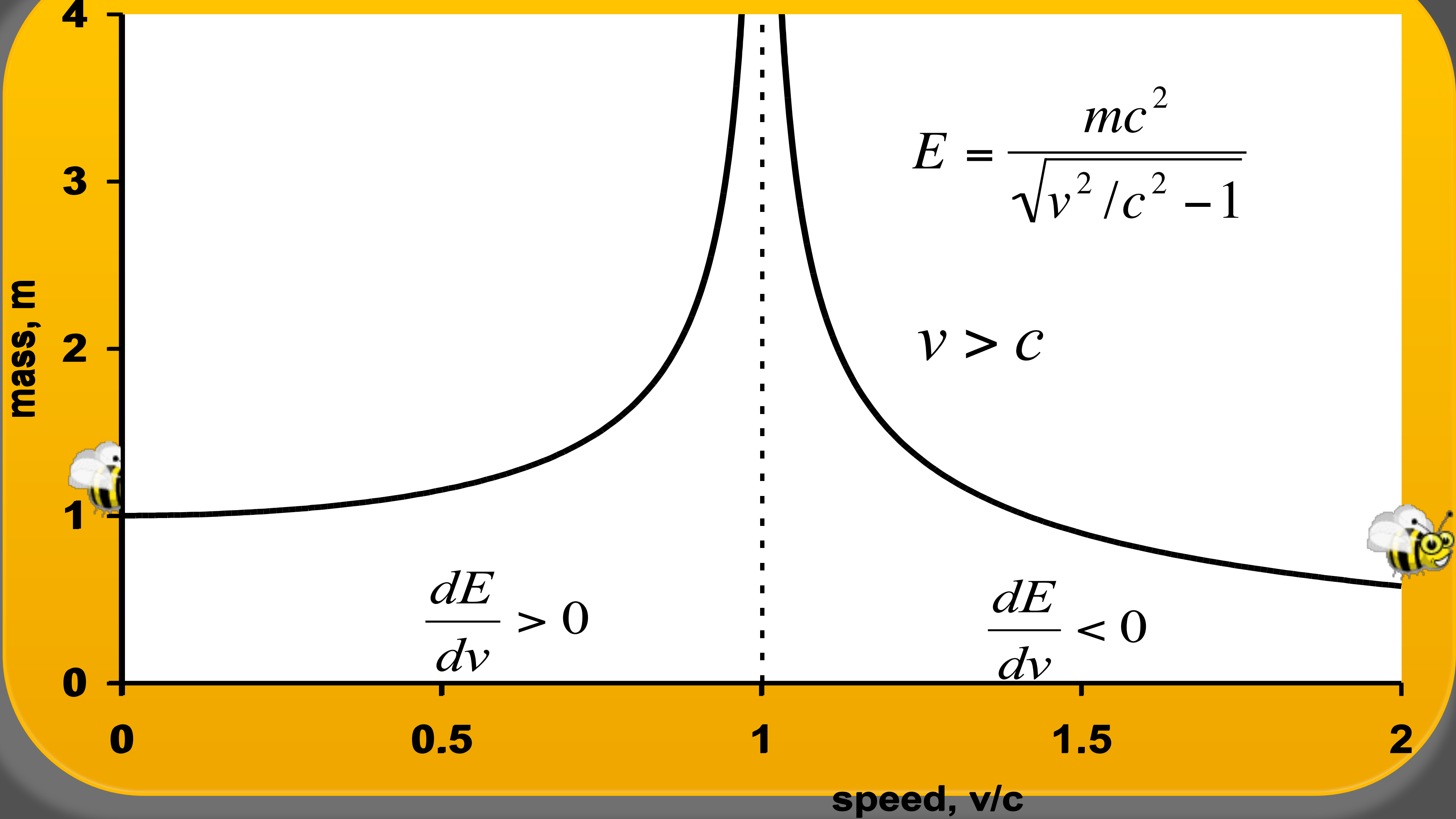
Is it possible that, although the OPERA
experimental claim has been refuted,
the neutrino could still be
infinitesimally superluminal?

If so, which physical
consequences would result?

Slide: Bob Ehrlich, George Mason

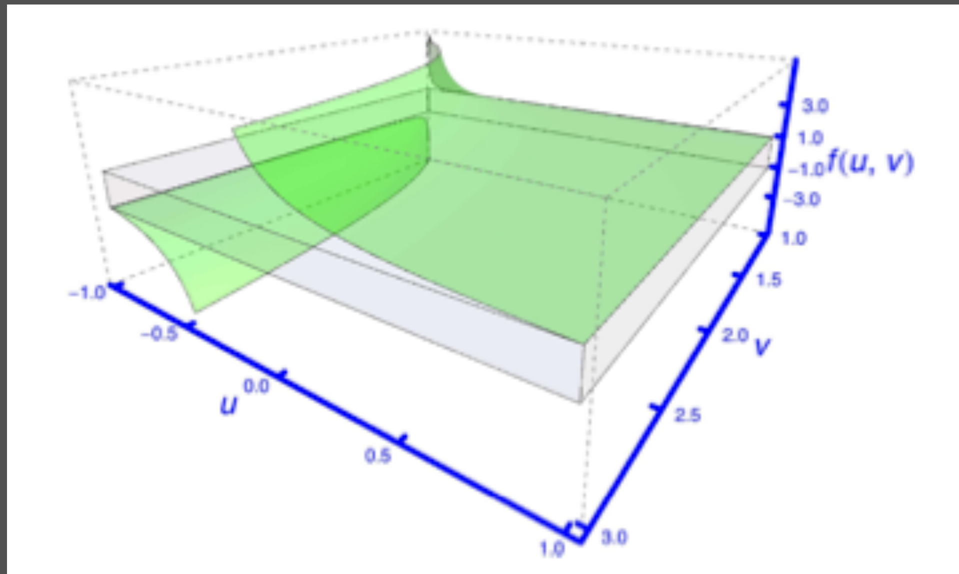
Why were tachyons first proposed?

$$E\text{^}2 - p\text{^}2 = -m^2 < 0$$



Bilaniuk, O.-M. P.; Deshpande, V. K.; Sudarshan, E. C. G. "'Meta' Relativity". *American Journal of Physics* **30** 718 (1962).

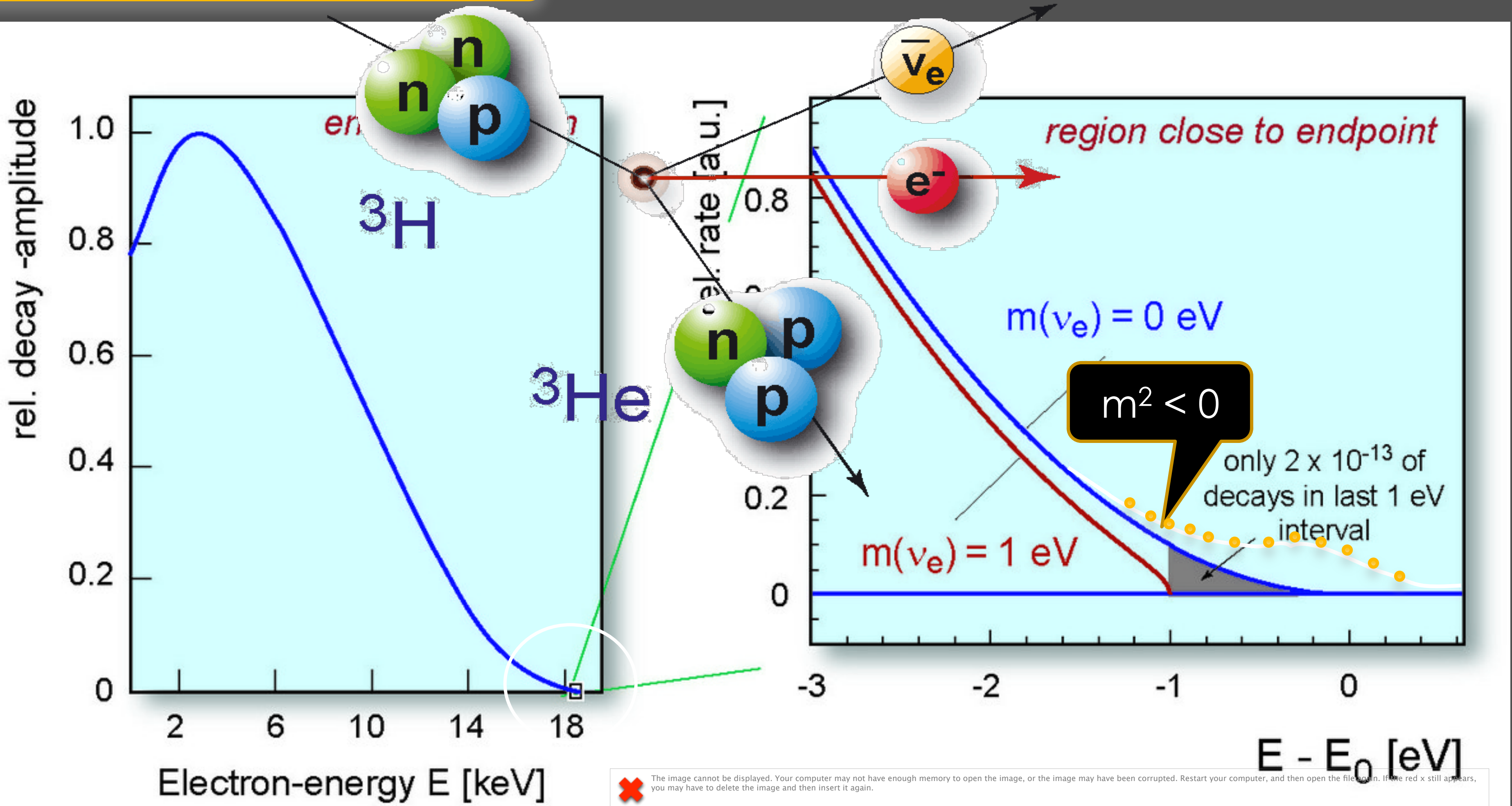
Superluminal particles remain
superluminal upon Lorentz transformation
(Einstein addition theorem remains valid)...



...and their existence is independent of the
axioms of special relativity (G. Szekely).

„Conjugate velocity" $u = -c^2/v < c$ for $v > c$.

Finding the mass of in tritium beta decay



Measured Mass Squares are All Negative...

Experiment	measured mass squared	formal limit	C.L.	Year
Mainz	-1.6 ± 2.5 ± 2.1	2.2	95 %	2000
Troitsk	-1.0 ± 3.0 ± 2.1 (**)	2.5	95 %	2000
Zürich	-24 ± 48 ± 61	11.7	95 %	1992
Tokyo INS	- 65 ± 85 ± 65	13.1	95%	1991
Los Alamos	- 147 ± 68 ± 41	9.3	95%	1991
Livermore	- 130 ± 20 ± 15	7.0	95%	1995
China	- 31 ± 75 ± 48	12.4	95%	1995
Average of PDG (98)	-27 ± 20	15	95 %	1998

http://cupp.oulu.fi/neutrino/nd-mass.html

It may not be (various citations)…

The $m^2(\nu_e)$-values obtained by the three checking experiments did agree among themselves within their combined error bars (compare figure 1); the error bars also excluded the ITEP-result. One could, however, recognize another problem which subsequently troubled the community for a long time: The mean values now fell into the unphysical negative region of the m^2-plot! Somehow the new experiments seemed to have overshot the mark. This feature was not significant for the Zürich-result where the error bar still extended into the positive sector. This also holds true for the results from the Tokyo- and Beijing-experiments, which were obtained from conventional spectrometers with modest luminosity [41,42].

But in view of the smaller errors, the results from Los Alamos, and particularly from Livermore, have a problem, caused, in all likelihood, by some unrecognised systematic error source. How should such a result be interpreted? Before 1998 one followed the so-called Bayesian approach, which was recommended by the Particle Data Group; it gave the following guidance : (i) The respective Gaussian error curve is centred at the place of the mean experimental value in the unphysical region and the fraction of its area which extends into the physically allowed region is determined; this fraction is considered the chance of the unphysical value found to be just a statistical fluctuation instead of being caused by some unrecognised systematic error. (ii) The residual area in the physically allowed sector is split into parts 95% to 5% (90% to10%) and the position of the split is considered the upper limit of the quantity in question with 95% (90%) confidence level (C. L.). Since 1998 the Particle Data Group favours the so-called frequentist approach [**43**], which gives similar results close to the physically allowed region.

It may not be…

4. Anomalious structures in the spectrum

Fitting of the first data of 1994 run with 4 basic variable parameters resulted in the value for m_ν^2 equal to $-22 \pm 5\,eV^2$ for the truncated spectrum with the truncation energy (further refered as E_{low}) more than $18300\,eV$. At lower truncation energy (down to $18000\,eV$) the m_ν^2 value increased to $-58\,eV^2$ and was accompanied by a strong increase in χ^2. The negative values for m_ν^2 obviously indicated that there exist some systematic effects not taken into account in the calculation of the theoretical spectrum [1].

Hmmm…..

4. Neutrino mass upper limit

Deduction of the neutrino mass from the data in presence of unexplained anomaly requires a special approach. As it was mentioned earlier the procedure adopted for this purpose consisted in addition to theoretical spectrum of the step function with two variable parameters supposing that such addition may describe in the first approximation local enhancement in the beta-spectrum near to the end-point. Distortion of beta-spectrum imitating the m_ν^2 effect should also be visible only near end point, otherwise the effect relatively rapidly sinks in growing statistical errors at increasing $E_0 - E$, but unlike the local enhancement it appears as an addition to (for negative m_ν^2) or deficiency (positive m_ν^2) of the spectrum intensity that is linearly increasing with $E_0 - E$. This difference allows one to separate both effects in fit procedure. Of course the size and position of the step being introduced as a free parameter, correlates with m_ν^2 and it increases the final error of neutrino mass thus acting as a kind of systematic error. This increase compensates main part of the uncertainty of substitution of a priory unknown anomaly shape by the step-like function. A possibility to distinguish neutrino mass effect from step strongly decreases with proximity of step position to end-point due to correlation of their parameters. Such correlation made impossible to use the data of Run97(1) and 98(1) for analysis on the neutrino mass in spite of their good statistics.

Normal (Tardyonic) Dirac Equation

$$\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ \mathbb{1}_{2\times 2} & 0 \end{pmatrix},$$

$$(\mathrm{i}\gamma^\mu \partial_\mu - m_1)\, \psi(x) = 0 \, .$$

Plane-Wave Solutions:

$$\psi(x) = U_{\pm}^{(1)}(\vec{k}) \, \exp(-\mathrm{i}k \cdot x) \, , \quad \phi(x) = V_{\pm}^{(1)}(\vec{k}) \, \exp(\mathrm{i}k \cdot x) \, ,$$

$$E^{(1)} = \sqrt{\vec{k}^2 + m_1^2}.$$

Eigenstates of Helicity [Non-Cyrillic]

$$a_+(\vec{k}) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) e^{i\varphi} \end{pmatrix}, \qquad a_-(\vec{k}) = \begin{pmatrix} -\sin\left(\frac{\theta}{2}\right) e^{-i\varphi} \\ \cos\left(\frac{\theta}{2}\right) \end{pmatrix},$$

$$\frac{\vec{\sigma}\cdot\vec{k}}{|\vec{k}|} a_\pm(\vec{k}) = \pm a_\pm(\vec{k}),$$

$$\sum_\sigma a_\sigma(\vec{k}) \otimes a_\sigma^+(\vec{k}) = \mathbb{1}_{2\times 2}, \qquad \sum_\sigma \sigma\, a_\sigma(\vec{k}) \otimes a_\sigma^+(\vec{k}) = \frac{\vec{\sigma}\cdot\vec{k}}{|\vec{k}|},$$

“Tardyonic” (“Normal”) Solutions and Sum Rules

$$U_+^{(1)}(\vec{k}) = \frac{(\not{k} + m_1)\, u_+(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} \sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}} a_+(\vec{k}) \\ \sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}} a_+(\vec{k}) \end{pmatrix},$$

$$U_-^{(1)}(\vec{k}) = \frac{(\not{k} + m_1)\, u_-(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} \sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \\ -\sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \end{pmatrix}.$$

$$E^{(1)} = \sqrt{\vec{k}^2 + m_1^2}.$$

$$V_+^{(1)}(\vec{k}) = \frac{(m_1 - \not{k})\, v_+(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} -\sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}}\, a_+(\vec{k}) \\ -\sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}}\, a_+(\vec{k}) \end{pmatrix}$$

$$V_-^{(1)}(\vec{k}) = \frac{(m_1 - \not{k})\, v_-(\vec{k})}{\sqrt{(E^{(1)} - |\vec{k}|)^2 + m_1^2}} = \begin{pmatrix} -\sqrt{\frac{E^{(1)} - m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \\ \sqrt{\frac{E^{(1)} + m_1}{2\,E^{(1)}}}\, a_-(\vec{k}) \end{pmatrix}$$

Well-Known Sum Rule:

$$\boxed{\sum_\sigma \mathcal{U}_\sigma^{(1)}(\vec{k}) \otimes \overline{\mathcal{U}}_\sigma^{(1)}(\vec{k}) = \frac{\not{k} + m_1}{2m_1},} \qquad \boxed{\sum_\sigma \mathcal{V}_\sigma^{(1)}(\vec{k}) \otimes \overline{\mathcal{V}}_\sigma^{(1)}(\vec{k}) = \frac{\not{k} - m_1}{2m_1}.}$$

Tachyonic Dirac Equation

$$\gamma^0 = \beta = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & -\mathbb{1}_{2\times 2} \end{pmatrix}, \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ \mathbb{1}_{2\times 2} & 0 \end{pmatrix},$$

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - \gamma^5 m\right) \psi(x) = 0.$$

[Chodos, Hauser, Kostelecky, Phys.Lett.B vol. 150 (1985), p.431]

$$\Psi(x) = U_\pm(\vec{k})\,\mathrm{e}^{-\mathrm{i}k\cdot x} \qquad \Phi(x) = V_\pm(\vec{k})\,\mathrm{e}^{\mathrm{i}k\cdot x},$$

$$E = \sqrt{\vec{k}^2 - m^2},$$

Tachyonic Solutions and Sum Rules

$$U_+(\vec{k}) = \frac{(\gamma^5\, m - \not{k})\, u_+(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} \sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \\ \sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \end{pmatrix},$$

$$U_-(\vec{k}) = \frac{(\not{k} - \gamma^5\, m)\, u_-(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} \sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_-(\vec{k}) \\ -\sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_-(\vec{k}) \end{pmatrix}.$$

$$V_+(\vec{k}) = \frac{(\gamma^5\, m + \not{k})\, v_+(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} -\sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \\ -\sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \end{pmatrix},$$

$$V_-(\vec{k}) = \frac{(-\not{k} - \gamma^5\, m)\, v_-(\vec{k})}{\sqrt{(E - |\vec{k}|)^2 + m^2}} = \begin{pmatrix} -\sqrt{\dfrac{|\vec{k}| + m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \\ \sqrt{\dfrac{|\vec{k}| - m}{2\,|\vec{k}|}}\, a_+(\vec{k}) \end{pmatrix}.$$

Important
Sum Rule (2012):

$$\boxed{\sum_\sigma (-\sigma)\, \mathcal{U}_\sigma(\vec{k}) \otimes \overline{\mathcal{U}}_\sigma(\vec{k})\, \gamma^5 = \frac{\not{k} - \gamma^5\, m}{2m},} \quad \boxed{\sum_\sigma (-\sigma)\, \mathcal{V}_\sigma(\vec{k}) \otimes \overline{\mathcal{V}}_\sigma(\vec{k})\, \gamma^5 = \frac{\not{k} + \gamma^5\, m}{2m}.}$$

$$\boxed{\sum_{\sigma}(-\sigma)\,\mathcal{U}_{\sigma}(\vec{k})\otimes\overline{\mathcal{U}}_{\sigma}(\vec{k})\,\gamma^{5}=\frac{\not{k}-\gamma^{5}\,m}{2m}\,,}$$

$$\boxed{\sum_{\sigma}(-\sigma)\,\mathcal{V}_{\sigma}(\vec{k})\otimes\overline{\mathcal{V}}_{\sigma}(\vec{k})\,\gamma^{5}=\frac{\not{k}+\gamma^{5}\,m}{2m}\,.}$$

Similar *Sum Rules* even hold for…

…TWO tardyonic mass terms…

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - m_1 - \mathrm{i}\gamma^5 m_2\right) \psi(x) = 0\,.$$

$$E = \sqrt{\vec{p}^{\,2} + m_1^2 + m_2^2}.$$

…TWO tachyonic mass terms…

$$\left(\mathrm{i}\gamma^\mu \partial_\mu - \mathrm{i}\, m_1 - \gamma^5 m_2\right) \psi(x) = 0\,.$$

$$E = \sqrt{\vec{p}^{\,2} - m_1^2 - m_2^2}.$$

[U.D.J. and B.J.Wundt, arXiv:1205.0521v3,
ISRN High-Energy Physics 2013 (2013) 374612]

Underlying Property...

...two tardyonic mass terms...
...Hermiticity...

$$H^{(t)} = \vec{\alpha} \cdot \vec{p} + \beta\, m_1 + \mathrm{i}\, \beta\, \gamma^5\, m_2 \,.$$

...two tachyonic mass terms...
...γ^5 Hermiticity ("Pseudo-Hermiticity")...

$$H' = \vec{\alpha} \cdot \vec{p} + \mathrm{i}\, \beta\, m_1 + \beta\, \gamma^5\, m_2 \,,$$

$$H' = \gamma^5 H'^{+} \gamma^5 .$$

[U.D.J. and B.J.Wundt, arXiv:1205.0521v3,
ISRN High-Energy Physics 2013 (2013) 374612]

Hermiticity versus Pseudo-Hermiticity

$$H^{(t)} = \vec{\alpha} \cdot \vec{p} + \beta\, m_1 + \mathrm{i}\, \beta\, \gamma^5\, m_2\,.$$

$$\begin{aligned} H^{(t)} &= \left(H^{(t)}\right)^+\,, \\ H^{(t)} &= \text{Hermitian operator} \end{aligned}$$

$$H' = \vec{\alpha} \cdot \vec{p} + \mathrm{i}\, \beta\, m_1 + \beta\, \gamma^5\, m_2\,,$$

$$\begin{aligned} H' &= \eta^{-1}\, H'^+\, \eta\,, \\ H' &= \gamma^5\, H'^+\, \gamma^5\,, \\ \eta &= \gamma^5\,, \\ \left(\gamma^5\right)^{-1} &= \gamma^5\,, \\ H' &= \text{pseudo-Hermitian (and that's enough!)} \end{aligned}$$

The Propagator in Field Theory...

$$\langle 0 \left| T\, \psi(x)\, \overline{\psi}(y) \Gamma \right| 0 \rangle = \mathrm{i}\, S(x-y)\,,$$

$$\psi(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{m}{E} \sum_{\sigma=\pm} \left\{ b_\sigma(k)\, \mathcal{U}_\sigma(\vec{k})\, \mathrm{e}^{-\mathrm{i}\, k\cdot x} + d_\sigma^{+}(k)\, \mathcal{V}_\sigma(\vec{k})\, \mathrm{e}^{\mathrm{i}\, k\cdot x} \right\}\,,$$

$$\begin{aligned} \{b_\sigma(k), b_\rho(k')\} &= \{b_\sigma^{+}(k), b_\rho^{+}(k')\} = 0\,, \\ \{d_\sigma(k), d_\rho(k')\} &= \{d_\sigma^{+}(k), d_\rho^{+}(k')\} = 0\,, \end{aligned}$$

$$\begin{aligned} \{b_\sigma(k), b_\rho^{+}(k')\} &= f(\sigma, \vec{k})\; (2\pi)^3\, \frac{E}{m} \delta^3(\vec{k} - \vec{k}')\, \delta_{\sigma\rho}\,, \\ \{d_\sigma(k), d_\rho^{+}(k')\} &= g(\sigma, \vec{k})\; (2\pi)^3\, \frac{E}{m} \delta^3(\vec{k} - \vec{k}')\, \delta_{\sigma\rho}\,, \end{aligned}$$

tardyonic choice: $\quad f(\sigma, \vec{k}) = g(\sigma, \vec{k}) = 1\,, \quad \Gamma = \mathbb{1}_{4\times 4}\,,$

tachyonic choice: $\quad f(\sigma, \vec{k}) = g(\sigma, \vec{k}) = -\sigma\,, \quad \Gamma = \gamma^5\,,$

$$\langle 0 | T\,\psi(x)\,\overline{\psi}(y)\Gamma | 0\rangle = \mathrm{i}\,S(x-y)\,,$$

$$\psi(x) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{m}{E}\sum_{\sigma=\pm}\left\{b_\sigma(k)\mathcal{U}_\sigma(\vec{k})\,\mathrm{e}^{-\mathrm{i}\,k\cdot x} + d_\sigma^+(k)\,\mathcal{V}_\sigma(\vec{k})\,\mathrm{e}^{\mathrm{i}\,k\cdot x}\right\},$$

$$\langle 0 | T\,\psi(x)\,\overline{\psi}(y)\gamma^5 | 0\rangle$$

We need the sum rule!!!

$$= \Theta(x^0-y^0)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{m}{E_k}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{m}{E_q}\sum_{\sigma=\pm}\sum_{\rho=\pm}\langle 0|b_\sigma(k)\,b_\rho^+(q)|0\rangle\,\mathcal{U}_\sigma(\vec{k})\otimes\overline{\mathcal{U}}_\rho(\vec{q})\,\gamma^5\,\mathrm{e}^{-\mathrm{i}\,k\cdot y}\,\mathrm{e}^{\mathrm{i}\,q\cdot y}$$

$$- \Theta(y^0-x^0)\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{m}{E_k}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\frac{m}{E_q}\sum_{\sigma=\pm}\sum_{\rho=\pm}\langle 0|d_\sigma(q)\,d_\rho^+(k)|0\rangle\,\mathcal{V}_\rho(\vec{k})\otimes\overline{\mathcal{V}}_\sigma(\vec{q})\,\gamma^5\,\mathrm{e}^{\mathrm{i}\,k\cdot y}\,\mathrm{e}^{-\mathrm{i}\,q\cdot y}$$

Again, we need the sum rule!!!

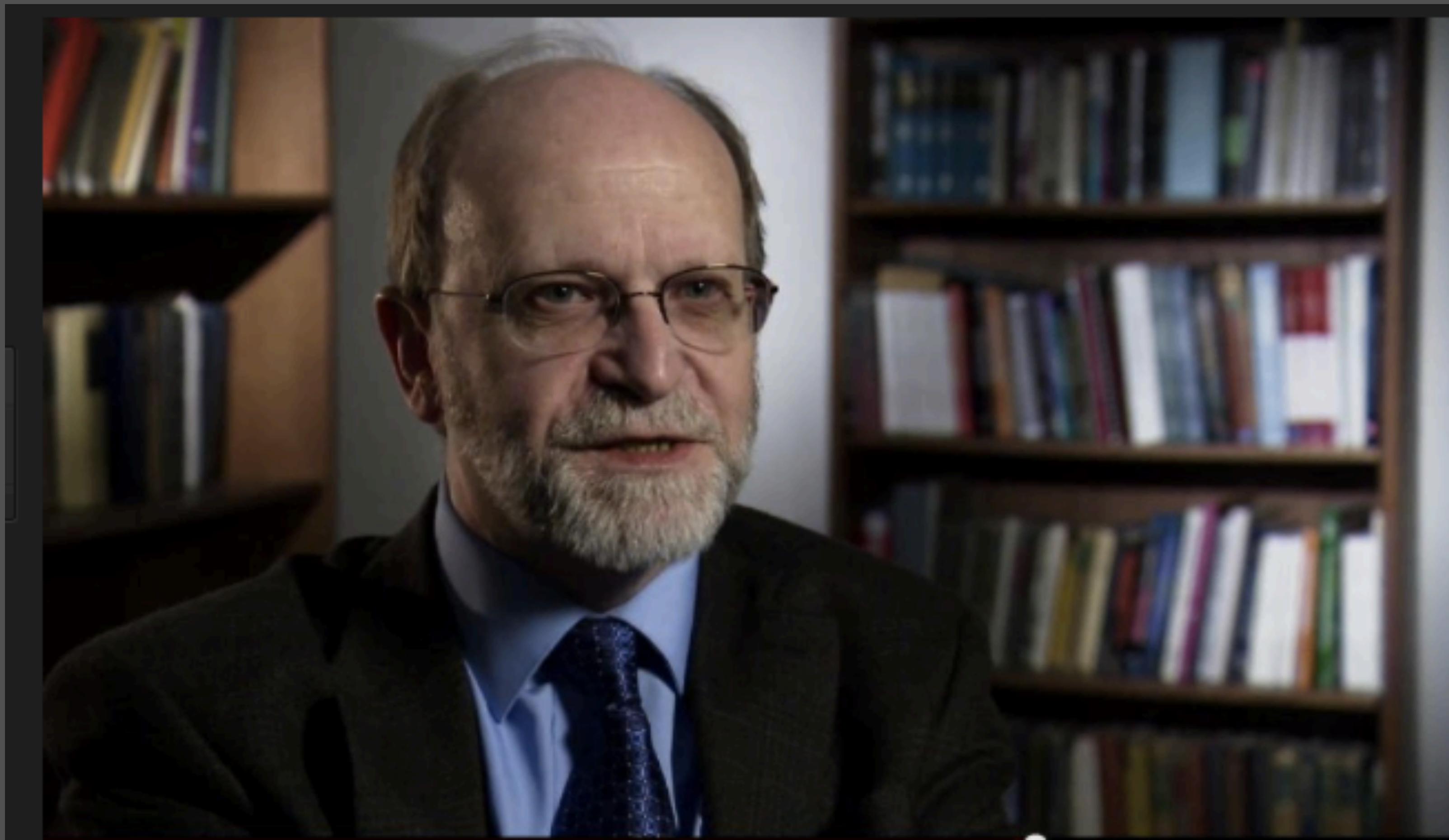
$$\langle 0 | T\,\psi(x)\,\overline{\psi}(y)\gamma^5 | 0\rangle = \mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot(x-y)}\,\frac{\not{k}-\gamma^5 m}{2m}\,\frac{m}{E_k}\,\frac{1}{k_0-E_k+\mathrm{i}\epsilon}$$

$$+\mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot(x-y)}\,\frac{-\not{k}+\gamma^5 m}{2m}\,\frac{m}{E_k}\,\frac{1}{k_0+E_k-\mathrm{i}\epsilon}$$

$$= \mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot(x-y)}\,\frac{\not{k}-\gamma^5 m}{2}\,\frac{1}{E_k}\left(\frac{1}{k_0-E_k+\mathrm{i}\epsilon}-\frac{1}{k_0+E_k-\mathrm{i}\epsilon}\right)$$

$$= \mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot(x-y)}\,\frac{\not{k}-\gamma^5 m}{k^2+m^2+\mathrm{i}\epsilon}$$

Mike Summers from George Mason said
something nice about this derivation,



The Propagator in Field Theory...

...tardyonic propagator...

$$S^{(1)}(k) = \frac{1}{\not{k} - m_1 + \mathrm{i}\epsilon} = \frac{\not{k} + m_1}{k^2 - m_1^2 + \mathrm{i}\,\epsilon}\,.$$

...tachyonic propagator...

$$S_T(k) = \frac{1}{\not{k} - \gamma^5\,(m + \mathrm{i}\,\epsilon)} = \frac{\not{k} - \gamma^5\,m}{k^2 + m^2 + \mathrm{i}\,\epsilon}\,.$$

[U.D.J. and B.J.Wundt, arXiv:1205.0521v3, ISRN High-Energy Physics, in press]

Negative Norm = Wrong Helicity...

tardyonic anticommutators:

$$\{b_\sigma(k), b^+_\rho(k')\} = (2\pi)^3 \, \frac{E}{m} \delta^3(\vec{k} - \vec{k}') \, \delta_{\sigma\rho} \,, \qquad \{d_\sigma(k), d^+_\rho(k')\} = (2\pi)^3 \, \frac{E}{m} \delta^3(\vec{k} - \vec{k}') \, \delta_{\sigma\rho} \,.$$

tachyonic anticommutators:

$$\{b_\sigma(k), b^+_\rho(k')\} = (-\sigma) \, (2\pi)^3 \, \frac{E}{m} \delta^3(\vec{k} - \vec{k}') \, \delta_{\sigma\rho} \,, \qquad \{d_\sigma(k), d^+_\rho(k')\} = (-\sigma) \, (2\pi)^3 \, \frac{E}{m} \delta^3(\vec{k} - \vec{k}') \, \delta_{\sigma\rho} \,.$$

$$|1_{k,\sigma}\rangle = b^+_\sigma(k)|0\rangle,$$

$$\langle 1_{k,\sigma}|1_{k,\sigma}\rangle = \langle 0 \,| b_\sigma(k) \, b^+_\sigma(k) |\, 0\rangle = \langle 0 \,| \{b_\sigma(k), b^+_\sigma(k)\} |\, 0\rangle = (-\sigma) \, V \, \frac{E}{m} \,,$$

Important: $\sigma = 1$ means right-handed-helicity neutrinos and left-handed antineutrinos for which the norm becomes negative...

$$\boxed{\langle\Psi|\psi_{\sigma=1}(x)|\Psi\rangle = \langle\Psi|\psi^{(-)}_{\sigma=1}(x) + \psi^{(+)}_{\sigma=1}(x)|\Psi\rangle = 0 \,,}$$

Negative Norm = Wrong Helicity...

One cannot reverse the suppression of the "wrong" helicity states because this would lead to contradiction with a smooth massless limit [see arXiv:1205.0521v3 for details]

See also an illustrative explicit calculation in arXiv:1201.6300 for the Dirac equation with imaginary mass, where the effect of an inversion of the imaginary mass term is studied.

Literature...

REVIEWS OF
MODERN PHYSICS
VOLUME 15, NUMBER 3 JULY, 1943

On Dirac's New Method of Field Quantization*

W. PAULI
Institute for Advanced Study, Princeton, New Jersey

As a generalization of the Hermitian conjugate operator, we introduce the adjoint operator which we denote by A^*. This is given by

$$A^* = \eta^{-1} A^{\dagger} \eta^{\dagger} = \eta^{-1} A^{\dagger} \eta, \tag{4}$$

$$\partial\psi/\partial t = -iH\psi,$$

hence

$$(\partial\bar{\psi}/\partial t)\eta = i\bar{\psi}H^{\dagger}\eta = i\bar{\psi}\eta H^*, \tag{6}$$

has to be self-adjoint,

$$H^* = H.$$

$\eta = \gamma^5$

REVIEWS OF
MODERN PHYSICS
VOLUME 15, NUMBER 3 JULY, 1943

On Dirac's New Method of Field Quantization*

W. PAULI
Institute for Advanced Study, Princeton, New Jersey

$$\frac{d}{dt}\int \bar{\psi}\eta\psi dq = \frac{d}{dt}\sum_{n,m} \bar{\psi}_n \eta_{nm} \psi_m$$

$$= i\bar{\psi}\eta(H^* - H)\psi = 0;$$

Is the existence of superluminal particles consistent with the axioms of special relativity?

[Gergely Szekely, Renyi Institute, of Mathematics, Budapest]

THE EXISTENCE OF SUPERLUMINAL PARTICLES IS CONSISTENT WITH THE KINEMATICS OF EINSTEIN'S SPECIAL THEORY OF RELATIVITY

GERGELY SZÉKELY

ABSTRACT. Within an axiomatic framework of kinematics, we prove that the existence of faster than light particles is logically independent of Einstein's special theory of relativity. Consequently, it is consistent with the kinematics of special relativity that there might be faster than light particles.

arXiv:1202.5790v1 [physics.gen-ph] 26 Feb 2012

There are a number of recent preprints on this subject...

paper submitted to J. Astropart. Phys.

The superluminal neutrino hypothesis and the ν mass hierarchy

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arXiv:1204.0484v8 [hep-ph] 26 Jul 2012

Published as [Astropart. Phys. 41 (2013) 1-6]

Volume 150B, number 6 PHYSICS LETTERS 24 January 1985

THE NEUTRINO AS A TACHYON

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and

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Received 30 October 1984

We investigate the hypothesis that at least one of the known neutrinos travels faster than light. The current experimental situation is examined within this purview.

IOP Publishing — Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **45** (2012) 444017 (19pp) doi:10.1088/1751-8113/45/44/444017

Pseudo-Hermitian quantum dynamics of tachyonic spin-1/2 particles

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DOI 10.1140/epjc/s10052-012-1894-4

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Localizability of tachyonic particles and neutrinoless double beta decay

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Received: 19 December 2011 / Revised: 9 January 2012

Neutrinos and Cosmology

Research Article

From Generalized Dirac Equations to a Candidate for Dark Energy

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Received 12 November 2012; Accepted 1 December 2012

Academic Editors: G. A. Alves, C. A. D. S. Pires, and F.-H. Liu

We consider extensions of the Dirac equation with mass terms $m_1 + i\gamma^5 m_2$ and $im_1 + \gamma^5 m_2$. The corresponding Hamiltonians are Hermitian and pseudo-Hermitian (γ^5 Hermitian), respectively. The fundamental spinor solutions for all generalized Dirac equations are found in the helicity basis and brought into concise analytic form. We postulate that the time-ordered product of field operators should yield the Feynman propagator ($i\epsilon$ prescription), and we also postulate that the tardyonic as well as tachyonic Dirac equations should have a smooth massless limit. These postulates lead to sum rules that connect the form of the fundamental field anticommutators with the tensor sums of the fundamental plane-wave eigenspinors and the projectors over positive-energy and negative-energy states. In the massless case, the sum rules are fulfilled by two egregiously simple, distinguished functional forms. The first sum rule remains valid in the case of a tardyonic theory and leads to the canonical massive Dirac field. The second sum rule is valid for a tachyonic mass term and leads to a natural suppression of the right-handed helicity states for tachyonic particles and left-handed helicity states for tachyonic spin-1/2 antiparticles. When applied to neutrinos, the theory contains a free tachyonic mass parameter. Tachyons are known to be repulsed by gravity. We discuss a possible role of a tachyonic neutrino as a contribution to the accelerated expansion of the Universe "dark energy."

[ISRN High-Energy Physics 2013 (2013) 374612]

Papers on Generalized Dirac Equations:

arXiv:1110.4171 (relativistic quantum mechanics)
[J.Phys.A 45 (2012) 444017]

arXiv:1201.0359 (quantized field theory)
[Eur.Phys.J C 72 (2012) 1894]

arXiv:1201.6300 (imaginary mass and helicity dependence)
[J.Mod.Phys. 3 (2012) 887]

arXiv:1205.0145 (attempt at neutrino mass running)
[Cent.Eur.J.Phys. 10 (2012) 749]

arXiv:1205.0521v3 (generalized theory and cosmology)
[ISRN High-Energy Physics 2013 (2013) 374612]

arXiv:1206.6342 (illustrative discussion)
[illustrative discussion and conference abstract]

Conclusions

Conclusions

(From arXiv:1205.0521v3)

On the other hand, if we assume that the neutrino is described by the tachyonic Dirac equation, then the following statements are valid:

- Statement #1: We can properly assign lepton number and use plane-wave eigenstates for incoming and outgoing particles, while allowing for nonvanishing mass terms and thus, mass square differences among the neutrino mass (not flavor) eigenstates.
- Statement #2: There is a natural resolution for the 'autobahn paradox' because a left-handed spacelike neutrino always remains spacelike upon Lorentz transformation and cannot be overtaken.
- Statement #3: The right-handed particle and left-handed antiparticle states are suppressed due to negative Fock-space norm.
- Statement #4: At least qualitatively, tachyonic neutrinos could yield an explanation for a repulsive force on intergalactic distance scales as they are repulsed, like all tachyons, by gravitational interactions ('dark energy').

A superluminal neutrino could appear to solve at least as many problems as it raises.

Conclusions

What happens if Dr. Spock overtakes a left-handed neutrino...

Spinor Sum for Positive-Energy States:

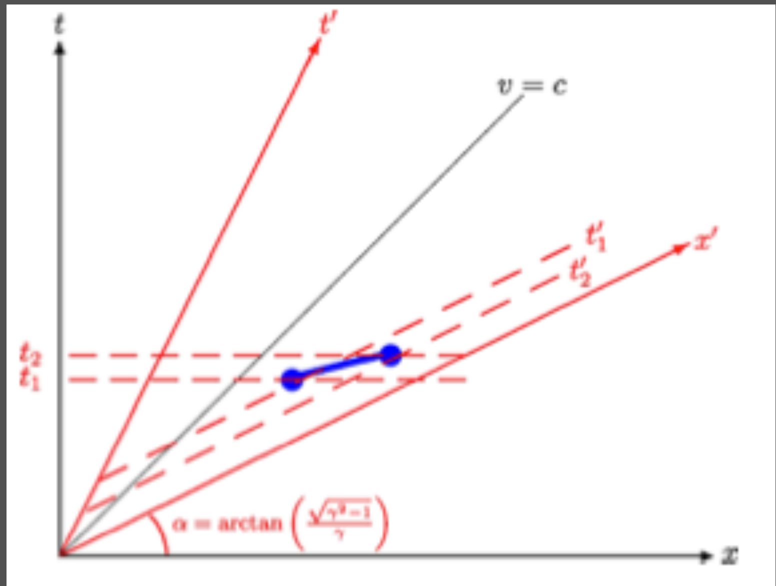
$$\boxed{\sum_{\sigma}(-\sigma)\, \mathcal{U}_{\sigma}(\vec{k}) \otimes \overline{\mathcal{U}}_{\sigma}(\vec{k})\, \gamma^{5} = \frac{\not{k} - \gamma^{5}\, m}{2m}}$$

Spinor Sum for Negative-Energy States:

$$\boxed{\sum_{\sigma}(-\sigma)\, \mathcal{V}_{\sigma}(\vec{k}) \otimes \overline{\mathcal{V}}_{\sigma}(\vec{k})\, \gamma^{5} = \frac{\not{k} + \gamma^{5}\, m}{2m}}$$

Thank you very much!

Reinterpretation Principle for the Quantum Theory



...We always have to reinterpret antiparticle trajectories by inverting the direction of time and space, but for superluminal particles, there is an additional difficulty because the time ordering of creation and annihilation may be reversed, depending on the velocity of the observer.

Possible solution offered in arXiv:1201.0359 [EPJC, 2012].