A GENERALIZED PROPORTIONATE ADAPTIVE ALGORITHM BASED ON CONVEX OPTIMIZATION

Jianming Liu, and Steven L Grant

Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, Missouri 65409.

ABSTRACT

A general framework is proposed to derive proportionate adaptive algorithms for sparse system identification. The proposed algorithmic framework employs the convex optimization and covers many traditional proportionate algorithms. Meanwhile, based on this framework, some novel proportionate algorithms could be derived too. In the simulations, we compare the new derived proportionate algorithm with the traditional ones, and demonstrate that it could provide faster convergence rate and tracking performance for both white and colored input in sparse system identification.

Index Terms — proportionate adaptive algorithm, echo cancellation, convex optimization

1. INTRODUCTION

The problem of acoustic echo cancellation (AEC) is usually approached by modeling the echo path impulse response with an adaptive filter and subtracting the estimated echo from the microphone output signal [1]. Due to the doubletalk and echo path change issues in AEC [2]-[3], the adaptive filter in AEC must be designed to converge fast and could track the echo path changes.

The design of adaptive filtering algorithm has been studied for quite a while, and the normalized least-mean-square (NLMS) algorithms are widely used [1]. Meanwhile, considering that the impulse response in echo cancellation is sparse, the family of proportionate algorithms exploits this sparseness to improve their performance [4]-[5]. In comparison to the NLMS algorithm, the proportionate NLMS (PNLMS) has very fast initial convergence and tracking when the echo path is sparse. The idea behind PNLMS is to update each coefficient of the filter independently of the others by adjusting the adaptation step size in proportion to the estimated filter coefficient [5]. However, the big coefficients converge very fast (the initial fast convergence) at the cost of slowing down dramatically the convergence of the small coefficients (after the initial period) [6]. As the large taps adapt, the remaining small coefficients adapt at a rate slower than NLMS. The mu-law PNLMS algorithm proposed in [6] addresses the issue of assigning too much update gain to large coefficients, which occurs in the proportionate algorithms. Furthermore, the \( l_0 \) norm family algorithms have recently drawn lots of attention for sparse system identification [8]-[10]. Therefore, an improved PNLMS algorithm based on the \( l_0 \) norm was proposed to represent a better measure of sparseness than the \( l_0 \) norm in PNLMS algorithm [11].

In this paper, we will propose a general framework of proportionate algorithm based on the convex optimization and sparseness measure. The proposed framework covers many traditional proportionate algorithms as in [5]-[7] and [11]. Meanwhile, based on this framework, some novel proportionate algorithms could be derived too.

This paper is organized as follows. Section 2 reviews the NLMS and traditional PNLMS algorithms. In Section 3, we present the proposed generalized proportionate algorithm based on convex optimization and sparseness measure. The simulation results and comparison to the previous algorithms are presented in Section 4. Finally conclusions are drawn in Section 5.

2. REVIEW OF NLMS AND PNLMS

The far-end signal \( x(n) \) is filtered through the room impulse response \( h(n) \) to get the echo signal \( y(n) \).

\[
y(n) = x(n)^* h(n) = x_n^T h_n
\]

where

\[
x_n = [x(n)x(n - 1)\cdots x(n - L + 1)]^T,
\]

\[
h_n = [h_0(n)h_1(n)\cdots h_{L-1}(n)]^T,
\]

and \( L \) is the length of echo path. This echo signal is added to the near-end signal \( v(n) \) (including both speech and background noise, etc.) to get the microphone signal \( d(n) \).

\footnote{This work is performed under the Wilkens Missouri Endowment.}

\footnote{Formerly Steven L Gay.}
\[ d(n) = x(n)^T h(n) + v(n) = x^T_n h_n + v(n). \] (2)

We define the error vectors as
\[ e(n) = d(n) - x^T(n) \hat{h}(n-1). \] (3)

The classical NLMS algorithm updates the filter coefficient as below [1]:
\[ \hat{h}(n) = \hat{h}(n-1) + \frac{x(n)e(n)}{x^T(n)x(n)} \] (4)

The classical PNLMS algorithm is as below [4]:
\[ \hat{h}(n) = \hat{h}(n-1) + \frac{G(n-1)x(n)e(n)}{x^T(n)G(n-1)x(n)} \] (5)

where
\[ G(n-1) = \text{diag}[\hat{h}_0(n-1), \ldots, \hat{h}_{l-1}(n-1)]. \] (6)

The mu-law PNLMS (MPNLMS) algorithm proposed in [6]-[7] used magnitude the logarithm of the coefficient magnitudes rather than using magnitudes directly as below:
\[ G(n-1) = \text{diag}[G(\hat{h}_0(n-1)), \ldots, G(\hat{h}_{l-1}(n-1))] \] (7)

and
\[ G(\hat{h}_k(n-1)) = \ln(1 + \mu |\hat{h}_k(n-1)|), \] (8)
in which \( \mu \) is a positive parameter. Based on the motivation that the \( l_0 \) norm can represent an even better measure of sparseness than the \( l_1 \) norm, an improved proportionate NLMS algorithm based on the \( l_0 \) norm was proposed as below [11]:
\[ G(\hat{h}_k(n-1)) = 1 - e^{-\sigma |\hat{h}_k(n-1)|} \] (9)

where \( \sigma \) is a positive parameter too. In next section, we will show that PNLMS, MPNLMS and \( l_0 \) norm PNLMS algorithms are all special cases of our proposed generalized proportionate algorithm scheme.

3. PROPOSED GENERALIZED PNLMS

The proposed generalized proportionate algorithm is derived as below. We try to minimize the following convex target with a constraint on linear system of equations:
\[ \min \quad F(\hat{h}(n)) \] (10)
subject to \[ d(n) = x^T(n) \hat{h}(n). \]

in which \( \hat{h}(n) \) is the correctness component as defined in [12] and \( F(\bullet) \) is a convex target function. The NLMS algorithm could be obtained by minimizing the following \( l_2 \) norm cost function:
\[ F(\hat{h}(n)) = \| \hat{h}(n) \|^2. \] (11)

For the sparse system identification, the traditional proportionate type algorithm was derived by optimizing (10) with the following \( l_1 \) norm
\[ F(\hat{h}(n)) = \| \hat{h}(n) \|. \] (12)

This is called from the basis pursuit perspective [12], and considering (6), (8)-(9), for classical PNLMS [4], we have
\[ G(\hat{h}_k(n-1)) = \hat{h}_k(n-1). \] (13)

Considering both (11) and (12), we propose to minimize the generalized convex target as below:
\[ F(\hat{h}(n)) = \frac{1}{2} x^T(n)G^{-1}(\hat{h}(n))\hat{h}(n) \] (14)
in which \( G(\hat{h}(n)) \) is a diagonal matrix defined as below:
\[ G(\hat{h}(n)) \approx \text{diag}[G(\hat{h}_0(n)), \ldots, G(\hat{h}_{l-1}(n))]. \] (15)

We propose to choose the \( G(\bullet) \) function as a class of sparseness measures satisfying the following Definition 1.

Definition 1. The function \( G : \mathbb{R} \to \mathbb{R} \) satisfies the following properties:
1) \( G(0) = 0 \), \( G(\bullet) \) is even and not identically zero;
2) \( G(\bullet) \) is non-decreasing on \([0, \infty )\);
3) The function \( t \mapsto G(t)|t| \) is non-increasing on \((0, \infty )\);

Therefore, considering the derivative of (14) is
\[ \frac{\partial F(\hat{h}(n))}{\partial \hat{h}(n)} = G^{-1}(\hat{h}(n))\hat{h}(n) \] (16)
\[ = \begin{bmatrix} \hat{h}_0(n) \\ \vdots \\ \hat{h}_{l-1}(n) \end{bmatrix} \begin{bmatrix} G(\hat{h}_0(n)) & \cdots & G(\hat{h}_{l-1}(n)) \end{bmatrix}^T, \]

and a differentiable function of one variable is convex on an interval if and only if its derivative is non-decreasing on that interval, we certify that (14) is a convex function.

Some commonly used sparseness measures are introduced in Table 1, and they are mainly from [13], but still included in this paper for completeness. The PNLMS, \( l_0 \) norm PNLMS and MPNLMS are included as No. 1, 3 and 4 separately. It should be noted that there are some other sparseness measure which do not fulfill the Definition 1, for example [14]. Meanwhile, we define
\[ \chi_r = \begin{cases} 1 & P \text{ true}; \\ 0 & P \text{ false}. \end{cases} \] (17)
Next we will derive the generalized proportionate algorithm through the similar approach in [12]. Using the method of Lagrange multiplier, the unconstrained cost function can be obtained as

\[ J(\tilde{h}(n)) = F(\tilde{h}(n)) + \lambda \left[ d(n) - x^T(n)\tilde{h}(n) \right]. \]  

(18)

where \( \lambda \) is a Lagrange multiplier. The derivative of the cost function with respect to the weight vector is

\[ \frac{\partial J(\tilde{h}(n))}{\partial \tilde{h}(n)} = G^{-1}(\tilde{h}(n))\tilde{h}(n) - \lambda x(n). \]

(19)

Setting the derivative equal to zero, we get

\[ \tilde{h}(n) = G(\tilde{h}(n))\lambda x(n). \]

(20)

Substituting (20) into constraint in (10), we get

\[ d(n) = x^T(n)G(\tilde{h}(n))\lambda x(n), \]

(21)

which means

\[ \lambda = \frac{d(n)}{x^T(n)G(\tilde{h}(n))x(n)}. \]

(22)

Substituting (22) into (20), we can obtain

\[ \tilde{h}(n) = \frac{G(\tilde{h}(n))x(n)d(n)}{x^T(n)G(\tilde{h}(n))x(n)}. \]

(23)

Similarly we approximate (23) with the adaptive filter of the previous time, which means

\[ G(\tilde{h}(n)) = G(\tilde{h}(n-1)) \]

\[ = \text{diag} \left[ G(\tilde{h}_0(n-1)) \ldots G(\tilde{h}_{L-1}(n-1)) \right]. \]

(24)

As mentioned in [12], the projection matrix of proposed algorithm is:

\[ P(n) = I - \frac{G(\tilde{h}(n-1))x(n)x^T(n)}{x^T(n)G(\tilde{h}(n-1))x(n)}. \]

(25)

The update equation for proposed generalized proportionate NLMS is:

\[ \tilde{h}(n) = P(n)\tilde{h}(n-1) + \tilde{h}(n), \]

(26)

which can be rewritten as

\[ \tilde{h}(n) = \tilde{h}(n-1) + \frac{G(\tilde{h}(n-1))x(n)e(n)}{x^T(n)G(\tilde{h}(n-1))x(n)}. \]

(27)

The equation (27) is the same as PNLMS in (5), but we have a generalized definition of \( G(\tilde{h}(n-1)) \) in (15).
4. SIMULATION RESULTS

In this section, we do computer simulations in the scenario of acoustic echo cancellation. We use a sparse echo path with length, \( L = 512 \), and the adaptive filter is with the same length. Parameter \( p \) is set to 0, and the parameters \( \sigma \) are set respectively so that they all contain the point \((0.9, 0.9)\). The convergence state of adaptive filter is evaluated with the normalized misalignment which is defined as

\[
10 \log_{10} \left( \frac{\| h - \hat{h} \|^2}{\| h \|^2} \right).
\]

In Fig. 3, the input is white Gaussian signal and independent white Gaussian noise is added to the system background with a signal-to-noise ratio, \( \text{SNR} = 30\text{dB} \). We compare proposed generalized proportionate algorithms in Table 1. Meanwhile, the simulations for colored input are shown in Fig. 4. The colored input signals are generated by filtering white Gaussian noise (WGN) through a first order system with a pole at 0.8. We could observe that the second sparseness measure with \( p = 0 \) outperforms the traditional PNLMS, mu-law PNMLS and \( l_0 \) norm PNLMS.

In Fig. 5 and Fig. 6, we compare the tracking ability of different proportionate algorithms for white and colored input separately, and we could draw the similar conclusion that the second sparseness measure could track the echo path change better than the other ones.

5. CONCLUSION

We have proposed a generalized proportionate adaptive algorithm based on the convex optimization and sparseness measures. The simulation results demonstrate that the new proportionate algorithm derived from our proportionate scheme outperforms the traditional PNLMS algorithms.
6. REFERENCES


