Fourier Series Table

Square Wave

\[ X_k = \begin{cases} 
-j \frac{2}{\pi k} & \text{for } k \text{ odd} \\
0 & \text{for } k \text{ even}
\end{cases} \]
Triangle Wave

\[ X_k = \begin{cases} 
  j \frac{4}{\pi^2 k^2} & \text{for } k = 1, 5, 9, \ldots \\
  -j \frac{4}{\pi^2 k^2} & \text{for } k = 3, 7, 11, \ldots \\
  X_{-k} & \text{for } k \text{ negative and odd} \\
  0 & \text{for } k \text{ even}
\end{cases} \]
Sawtooth

\[ X_k = \begin{cases} 
  \frac{j}{\pi k} & \text{for } k \neq 0 \\
  0 & \text{for } k = 0 
\end{cases} \]
Full Wave Rectified Sine

\[ X_k = \frac{-2}{\pi \left( 4k^2 - 1 \right)} \]
Half Wave Rectified Sine

\[ X_k = \begin{cases} 
\frac{-1}{\pi (k^2 - 1)} & \text{for } k \text{ even (including 0)} \\
\frac{j}{4} & \text{for } k=1 \\
\frac{j}{4} & \text{for } k=-1 \\
0 & \text{for } k \text{ odd (except for +1 and -1)} 
\end{cases} \]
Problem 5 – Corn

There’s an old saying that if the weather conditions are favorable, on a quiet night one can hear corn plants grow. I want to test this by putting an audio recorder in the middle of a corn field at night. I expect the loudest sound will be the crickets chirping, at 30 dBSPL. The sound of the corn, if it exists, may be so quiet that humans can barely hear it. My audio recorder has a microphone with a sensitivity of 300 mV/Pa, and it is followed by an amplifier with gain $G$. I want to make $G$ as large as possible, but I never want to clip the signal. The amplifier is connected to a power supply of +/- 3 volts. I don’t know the crest factor of growing corn, but the crest factor of crickets chirping is about 15.

What is the largest gain, $G$, I can use for this application?
Problem 6 – Extension Cord

An audio amplifier says it can deliver 50 watts of power into an 8 ohm speaker. This means the amplifier can generate a sinusoid that when connected to the speaker, will cause the speaker to dissipate 50 watts of power. However my speaker is 100 feet away from my amplifier. I have a 100 foot extension cord, made of 18 gauge wire. A table I found on-line says 18 gauge copper wire has a resistance of 6 ohms per 1000 feet. Answer both of the following questions, showing your work for full credit

a) If I make no changes to the audio amplifier, what power will the speaker see when it is connected to the amplifier through the extension cord?

b) If your answer to part a is less than 50 watts, can I get it back to 50 watts just by turning up the volume control a little more? Justify your answer, don’t just answer yes or no.
Problem 7 – Op Amps

A circuit has two inputs, x(t) and y(t). The circuit has one output, z(t). Sketch an op-amp circuit that will produce the following

\[ z(t) = 2x(t) - \frac{d}{dt}(3x(t) - 4y(t)) \]

I want x(t) to see an input impedance of 1 kΩ, y(t) to see an input impedance of 2 kΩ, and I want the output z(t) to have an output impedance of 3 Ω.
Problem 8 – Fundamental Frequency

Find the fundamental frequency of the signal shown below

\[ x(t) = 25.3 \cos(2\pi 29.393t - 99.222) - 44 \sin(2\pi 355.81t + 88.2) + 76.76 \cos(2\pi 96.577t) \]