There are the units used below, and the abbreviations for each.

Energy is measured in Joules (J)
Power is measured in Watts (W)
Voltage is measured in Volts (V)
Current is measured in Amperes (A)
Force is measured in Newtons (N)
Distance is measured in meters (m)
Time is measured in seconds (s)
Mass is measured in kilograms (kg)
Pressure is measured in Pascals (Pa)
Resistance is measured in ohms (Ω)
Electric Charge is measured in Coulombs (C)

Energy is usually the product of two things. If you are pushing a rock, it is how hard you have to
lean on the rock, times the distance you move it (force * distance). If you are pumping up a car
tire, it is the amount you raise the pressure times the size of the tire (pressure * volume). If you
are pushing electrons around a circuit, then it is the voltage you apply times the number of
electrons you move (voltage times charge). Using the standard units, one can define a Joule as

\[ J = N \cdot m = Pa \cdot m^3 = C \cdot V \]

We often want to know how fast energy is moving from one point to another. The rate of flow
of energy is of course called power. Using the standard units, one can define a Watt as

\[ W = \frac{J}{s} = \frac{N \cdot m}{s} = \frac{Pa \cdot m^3}{s} = \frac{C \cdot V}{s} = A \cdot V \]

To find the power dissipated by an electrical circuit, you normally need to know voltage and
current. However for a circuit that just has a resistor in it, the current is proportional to voltage,
which of course we know as Ohm’s law

\[ V = A \cdot \Omega \]

Gasses, like air, have a property that is roughly equivalent to ohm’s law. Air particles move
from regions of high pressure to regions of low pressure. The speed at which the particles move
depends on the pressure difference, and also on something called the acoustic impedance, z, of
the gas. This makes the gas version of ohm’s law
Pressure = velocity * acoustic_impedance

To get the units to work out properly, acoustic impedance needs units of Pascals seconds per meter

\[ Pa = \frac{m}{s} \cdot \frac{Pa \cdot s}{m} \]

The value of \( z \), the acoustic impedance, depends on the type of gas, its temperature, the atmospheric pressure, and a bunch of other things. But for air at room temperature air at one atmosphere, the value of \( z \) is about 400. So it takes about 400 Pa of pressure difference to get air to move at 1 meter/sec.

Let’s now look at the mechanical part of a speaker. It has a cone that moves back and forth. When this cone moves, it applies a force to the air around it. Call this force \( f \) Newtons. The cone also moves some distance while applying this force. Call that distance \( x \) meters. So the energy the cone puts into the air is then

\[ \text{SoundEnergy} = fx \cdot N \cdot m = fx \cdot J \]

To find the power of the sound wave coming off the speaker, just divide the energy by time. If it takes the speaker \( t \) seconds to move the \( x \) meters I mentioned above, then the power in the resulting sound wave, in watts, is:

\[ \text{SoundPower} = \frac{\text{SoundEnergy}}{\text{sec}} = \frac{fx \cdot J}{t} = \frac{fx}{t} \cdot W \]

The electrical system will have to push at least that many watts into the speaker. Actually speakers are horribly inefficient, so it will take a lot more electrical power than what I showed above. But if a speaker was 100% efficient, the number I calculated above is how much electrical power would have to go into the speaker.

The sound wave created by the speaker moves away from it, carrying its power with it as it goes. Real world speakers are directional, meaning they send more power in some directions than others. Also in the real world the power of the sound wave will be absorbed (turned into heat) by the air and objects that the wave hits. We are going to ignore all those effects. We will assume the sound power radiates equally well in all directions – and none of its power is absorbed or lost.

So after the sound wave has moved \( d \) meters, all the power is on the surface of a sphere of radius \( d \). The surface area of this sphere is

\[ 4\pi d^2 \cdot m^2 \]

So the power density of the sound, or the power per square meter, is now...
\[
\text{SoundPowerDensity} = \frac{\text{SoundPower}}{\text{Area}} = \frac{xf}{4\pi d^2} \frac{W}{m^2}
\]

The sound power density is what controls how loud a microphone, or our ears, think a sound is. A microphone, or our ears, will not collect all the sound power produced by a speaker. The microphone/ear will collected power over a fixed area. The total power collected will be the area of the microphone sensor or ear, times the sound power density.

As we move away from a speaker, the sound power density decreases with the distance squared.

Microphones convert pressure changes into voltages. So let’s re-write the sound power density in terms of pressure. First we use the fact that a Watt is a Newton*meter/second

\[
\text{SoundPowerDensity} = \frac{xf}{4\pi d^2} \frac{W}{m^2} = \frac{xf}{4\pi d^2} \frac{N \cdot m}{s \cdot m^2}
\]

And a Pascal is a Newton per square meter, so

\[
\text{SoundPowerDensity} = \frac{xf}{4\pi d^2} \frac{N \cdot m}{s \cdot m^2} = \frac{xf}{4\pi d^2} \frac{Pa \cdot m}{s \cdot m^2}
\]

This makes the sound power density a pressure times a velocity. Earlier I mentioned that in simple gas models, the pressure difference and velocity are related by the acoustic impedance, \(z\) (just as voltage and current are related by the electrical impedance). Let’s multiply both sides of our equation by the acoustic impedance, which has units Pascals seconds / meter.

\[
\text{SoundPowerDensity} \cdot z = \frac{xfz}{4\pi d^2} \frac{Pa \cdot m}{s} = \frac{xfz}{4\pi d^2} \frac{Pa \cdot s}{m}
\]

This shows that the square of the pressure is proportional to the sound power density. Since microphones convert pressure to voltage, this means the voltage squared at the microphone output will be proportional to the sound power density. If you measure the pressure in Pascals, and you want sound power density in watts per square meter, then you need to divide the pressure squared by the acoustic impedance (about 400).

Notice that sound power density drops with distance squared, and sound power density is proportional to pressure squared, so that means that pressure drops linearly with distance.

When measuring sound, we normally do not use units of Watts or Watts per square meter. There are a number of reasons for this. Part of it is that the power and power density is usually a very small fraction of a watt, or a small fraction of a watt per square meter. Another reason is that we often have a very wide range of values we are interested in measuring. A human whispering will generate about 10 pW of sound power, while a turbojet engine generates about 100 kW of acoustic power. By the way, 100 kW is about 130 horsepower. A significant amount of power is consumed in a turboprop engine, just to make noise, and that 130 horsepower does nothing to move the airplane forward.
Rather than using watts, or watts per square meter, we use a log scale for both sound power and sound power density. We will do sound power first.

If the sound power generated by an acoustic source is $P$ watts, then we say the Sound Power Level, or Sound Watt Level, or SWL, or Acoustic Power level, or $L_W$ is

$$L_W = 10 \log_{10}\left(\frac{P}{10^{-12}}\right)\text{dBSWL}$$

Since most sounds of interest to microphones have a power level over 1 pW, the $L_W$ number will normally be positive. Notice that when people use this term, they try to get a “W” in there somewhere. That is to remind you that we are dealing with “Watts” as opposed to “Watts per square meter”.

Our ears are more sensitive to some frequencies than others. So when calculating the total power produced by a sound source, people sometimes weight, or scale, or emphasize, frequencies based on the response of a typical human ear. There are different ways to emphasize frequencies, but a common one is called A-weighting (and oddly enough, there is a B, C and D weighting also). If someone is emphasizing frequencies based on this A weighting, they will change $L_W$ to $L_{WA}$.

Now let’s look at how we measure the sound power density. If at a particular location the sound power density is $D$ watts per square meter, we describe this using the Sound Pressure Level, or SPL, or $L_p$.

$$L_p = 10 \log_{10}\left(\frac{D \cdot z}{4 \cdot 10^{-10}}\right)\text{dBSPL}$$

Where $D$ is the sound power density in watts per square meter, and $z$ is the acoustic impedance of the air, which is approximately 400 for room temperature and one atmosphere. Since $D$ times $z$ is just equal to the pressure in pascals squared, we can rewrite this as

$$L_p = 10 \log_{10}\left(\frac{p_{rms}^2}{4 \cdot 10^{-10}}\right)\text{dBSPL}$$

Where $p_{rms}$ is the rms of the pressure of the sound signal (the square root of the average of the square of the pressure). It is also common to bring the square out in front, to get

$$L_p = 20 \log_{10}\left(\frac{p_{rms}}{2 \cdot 10^{-5}}\right)\text{dBSPL}$$

Since the rms pressure of the quietest sound a human can hear is about $2 \times 10^{-5}$ Pascals, $L_p$ is usually a positive number.

Microphone Sensitivity
Microphone produce a voltage that is proportional to pressure. There is a conversion factor for the microphone, which will tell you how many volts you will see at the output for each 1 Pascal change in the air pressure. Usually you will see a small fraction of a volt change for each Pascal. So for example a microphone might generate 10 mV/Pa. The convention is to specify microphone sensitivity using a dB scale also. So if a microphone produces x volts for each pascal of pressure change, we will say its sensitivity is

$$\text{Microphone sensitivity} = 20 \log_{10} \left( \frac{x \text{ dBV}}{\text{Pa}} \right)$$

It is often convenient to go from L1 dBSPL at the microphone input, to L2 dBW into a 1 ohm load, at the microphone output using a microphone with a sensitivity of L3 dBV/Pa. The calculation would be approximately

$$L_2 \text{ dBW} = L_1 \text{ dBSPL} - 94 + L_3 \text{ dBV/Pa}$$

For most audio applications L1 will be a positive number, often in the range of 0 to 100. The L3 number will usually be negative, in the range of a few tens of dB.

**Speaker Sensitivity**

Since speakers take in electrical power, and put out acoustic power, you might think there would be a conversion factor to describe how efficient they are. And there may be, but people usually don’t use it. What they use is something called speaker sensitivity – which relates electrical power in, to sound power density at a particular location. The sensitivity of a speaker will be the dBSPL level you measure 1 meter from the speaker when you put in 1 watt of electrical power.

For example, if a speaker has a sensitivity of 80 dB, then when you put in one watt of electrical power, you get 80 dBSPL at one meter.

If you put in 100 watts of electrical power, the sound power density at 1 meter will be 100 times as high. Remember the definition of SPL was

$$L_p = 10 \log_{10} \left( \frac{D \cdot z}{4 \cdot 10^{-10}} \right) \text{ dBSPL}$$

So if D is 100 times larger, then $L_p$ will be 20 dB larger because

$$10 \log_{10} \left( \frac{100 \cdot D \cdot z}{4 \cdot 10^{-10}} \right) = 10 \left[ \log_{10} (100) + \log_{10} \left( \frac{D \cdot z}{4 \cdot 10^{-10}} \right) \right]$$

$$= 10 \left[ 2 + \log_{10} \left( \frac{D \cdot z}{4 \cdot 10^{-10}} \right) \right]$$

$$= 20 + \log_{10} \left( \frac{D \cdot z}{4 \cdot 10^{-10}} \right)$$

So it is easier to just describe the electrical input power in dBW, because then at 1 meter the sound power density level is just the electrical power in dBW plus the speaker sensitivity in dB.
As you move away from the speaker, the sound power density drops with distance squared.

\[ L_p = 10 \log_{10} \left( \frac{D \cdot z}{d^2 \cdot 4 \cdot 10^{-10}} \right) = 10 \log_{10} \left( \frac{D \cdot z}{4 \cdot 10^{-10}} \right) - 20 \log_{10} (d) \]

Putting all this together, if you put in \( L_1 \) dBW of electrical power into a speaker with a sensitivity of \( L_2 \) dB, and you measure the sound power density while standing \( d \) meters from the speaker, you will see \( L_3 \) dBSPL where

\[ L_3 \text{dBSPL} = L_1 \text{dBW} + L_2 \text{dB} - 20 \log_{10} (d) \]

The absolute efficiency of a speaker is hard to calculate, as the sound power does not radiate equally in all directions. But just to give you a rough idea of where things stand, suppose a speaker had a sensitivity of 80 dB. The power density in Watts per square meter, at a distance of 1 meter from the speaker is

\[ 80 = 10 \log_{10} \left( \frac{D \cdot z}{4 \cdot 10^{-10}} \right) \]

Using an acoustic impedance, \( z \), of 400, we get the power density, \( D \), is about 0.1 mW per square meter. A sphere of radius 1 meter has an area of about 12.5 square meters, so the total acoustic power would be on the order of 1 mW. We put in 1 watt of electrical power to get out 1 mW of acoustic power. That makes the speaker 0.1% efficient. The other 99.9% of the input power went into heat. This speaker is a room heater which happens to make a little bit of noise.