Product Upgrades Based on Minimum Expected Quality Loss

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Abstract. The need to maintain or sustain a product's usefulness to or beyond its initial lifecycle expectancy is often driven by investment costs, market share, sales revenues, new product release schedules, standardization, uninterruptable usage, or declining budgets. A question then arises as to how often to upgrade a product. Aside from these typically discussed rationales, the paper concludes that a viable approach is to consider the upgrade cycle that results in the minimum cost based on quality. A general quality loss function was developed to investigate the expected quality loss for quality characteristics such as nominal-the-best, smaller-the-best, and larger-the-better. It is shown that when a shape parameter is introduced into the loss function, product upgrade cycles can be managed to minimize the expected losses.

Introduction

Maintenance and sustainment costs are typically one-third of the development costs, as was briefed to the Government Accountability Office (Chaplain 2008) for the Space Shuttle, to 70% of the lifecycle costs for the general category of software (Boehm 2001). Often, managers responsible for maintenance and sustainment target costs reductions on the order of 15% to 20% to improve product profitability. Perhaps such actions assume that customers are pleased by both the gesture to reduce costs and the company's interests in supporting fielded products. However for customers, perhaps the most meaningful consideration of continued product support is lower cost of ownership.

The authors posit that the upgrade cycle for fielded products could be based on the expected quality loss that results from the period of the upgrade. The consequence would be a Pareto-efficient determination of the upgrade period. To achieve Pareto-efficiency (based on the principle that one-sided benefit to a party to a negotiation results in an inequitable distribution of losses), losses for all stakeholders must be considered and incorporated into a cooperative exchange of benefits and losses. A common distinction between the interests of stakeholders can be depicted graphically as leaning towards either smaller or larger than some position that will eventually be the negotiated settlement. That is, the agreement between two stakeholders is defined as the position whereby neither side to a negotiation has an unfair or disproportionate advantage. For the purpose of this paper, the mathematics simplifies by assuming an idealized negotiation (Figure 1) where two parties incur equal losses about a center point target value, m.

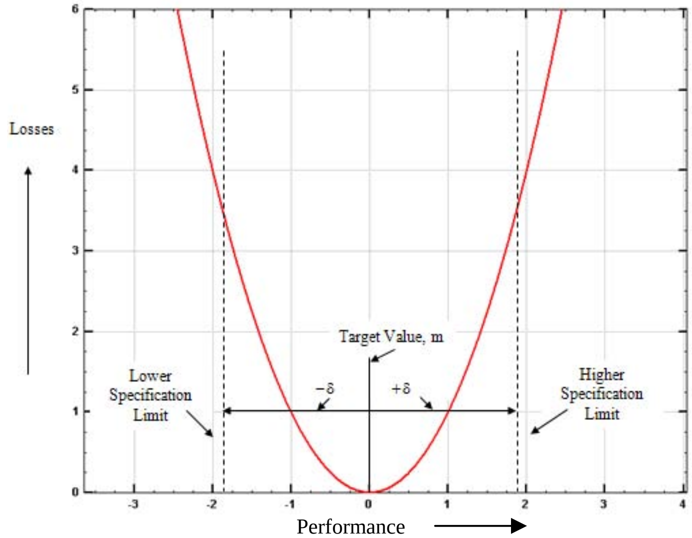

Figure 1. Pareto Efficient Negotiation

The minimum loss depicted as the quality-loss function in Figure 1 defines the *target value* of the critical performance characteristic, *m*, as a negotiation between two strategies of quality. One party to the negotiation determines that more performance is better (considered as larger-the-better (LTB) strategy) while the other party considers smaller-the-better (STB) demands on performance is required. LTB performance means that the function being performed derives benefits from a lower loss at larger values of performance, *m*. Alternatively, STB performance signifies a lower loss is expected with a smaller performance requirement, *m*. For example, a seller might want to deliver more product performance but is unwilling to accept increased costs which may lead to reduced marketshare, while the buyer might expect more product performance for lower costs. Figure 2 illustrates LTB and STB plotted with the x-axis as performance and the y-axis as the loss.

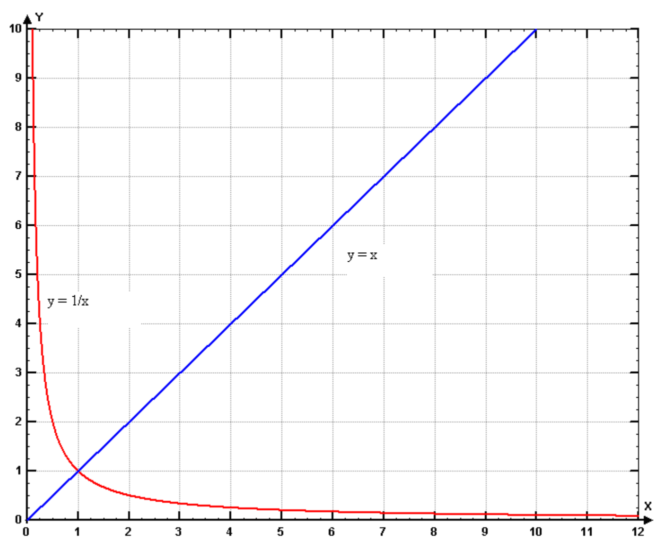

Figure 1. Smaller-the-Better (STB) and Larger-the-Better (LTB)

Combining the two curves, x and $1/x$, results in a pictorial representation of negotiation. Figure 3 shows the resultant curve. The competition between one party espousing STB and another party posturing LTB is in essence a negotiation that results in defining a working regime that reflects the interests, needs, and requirements of both parties to the negotiation. The property of Pareto-efficiency (that one-sided benefit to a party to a negotiation results in an inequitable distribution of losses) should guide the selection and agreement of m. The result of a Pareto-efficient determination of m is a minimum loss for that negotiation, Figure 3.

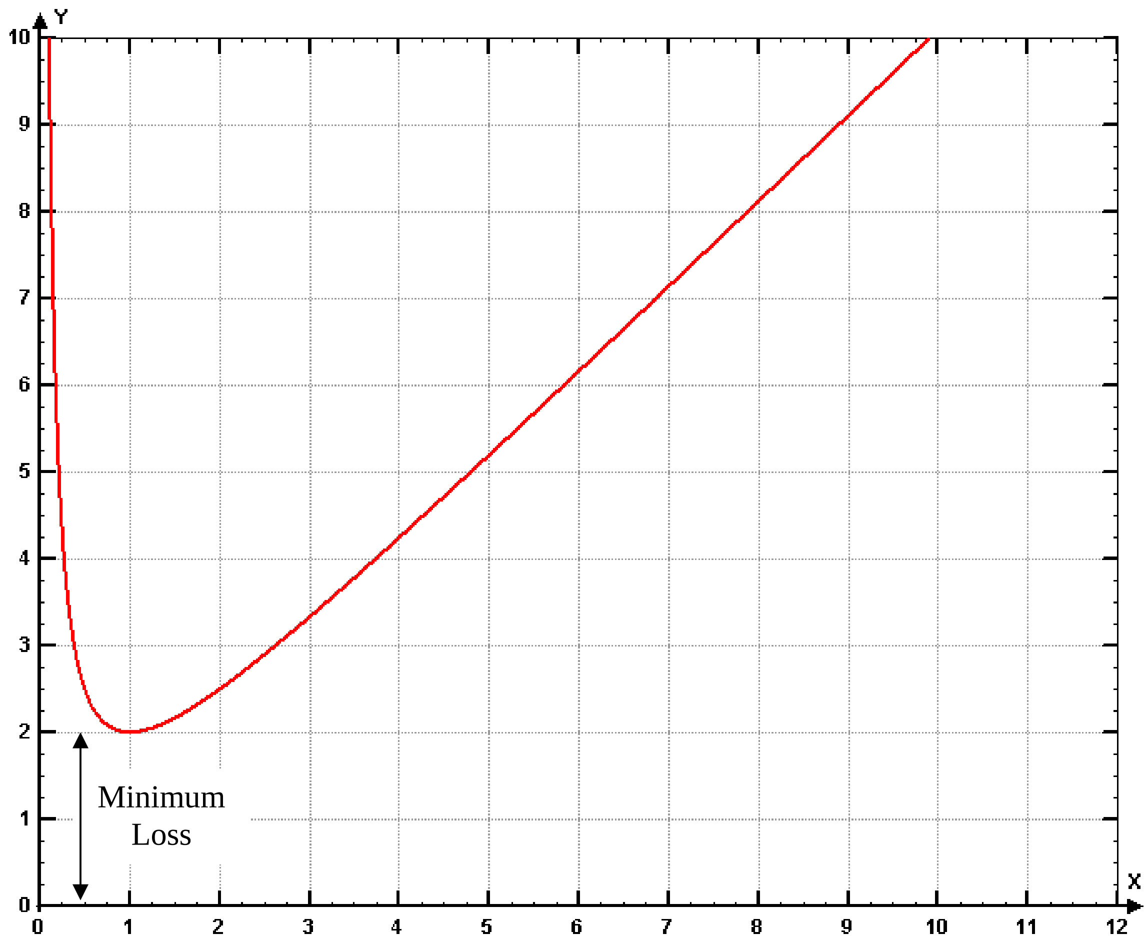

Figure 2. Combining Two Loss Distributions that Compete for a Definitive Product Upgrade Period, m

From Figure 3, the resultant quality-loss distribution has a minimum at $m = 1$, representing the minimum loss that can be caused after the upgraded product is shipped. This minimum loss represents the most effective periodicity for the product upgrade within the product's specified environment, given the conflicting constraints of STB and LTB. The goal of the stakeholder negotiations is to minimize the total system losses due to and during the course of upgrading and releasing upgraded products.

Losses to the customers result from an early release of an upgraded product that may not take full advantage of better technology. This may manifest itself through lower performances of product functions. Later release of an upgrade product may deprive customers of productivity that could have been made given earlier release of an upgrade. Premature release of an upgraded product may require fixes and patches to achieve operational effectiveness; while perfectly functioning upgrades may be function rich, but performance poor. The result of an early or late release is in effect to slide the performance parameter horizontally in Figure 3. Therefore the resultant quality loss function only has a minimum value when Pareto-efficiency is achieved.

Outline of the General Quality Loss Function

Quality Characteristics. To achieve the desired level of quality and to determine the target value for upgrading a product, stakeholders pose the following question – how much loss can I incur? To answer this question, a general quality loss function must be developed. We introduce a shape parameter that governs the amount of losses as a function of the periodicity, m, and present a function which covers nominal-the-better (NTB).

Traditionally, quality is viewed as a step function such as a good product or a bad product. This view assumes a product quality is uniformly good between the lower specification and the upper specification. Sometimes traditional decision makers and those using Taguchi's loss function will make the same judgments. If organizations consider both the position of the average and the variance, and if the averages are equal and/or the variances are equal, then the traditional decision maker and one using Taguchi's loss function will make the same decision. Typically however, the traditional decision maker calculates the percentage of defective units over time, when both the average and variance are different. Both the average performance and variation from a target value are measures of quality (Taguchi G., et. al. 1989).

Taguchi believes that the customer becomes increasingly dissatisfied as performance departs farther away from the target value for performance of a function. His extensive work with industry suggests a quadratic curve best represents a customer's dissatisfaction with a product's performance. The first derivative of a Taylor Series expansion taken about the target value is a quadratic curve when the target value is set to zero. The curve's minimum is centered on the target value, which Taguchi, et. al. (1989) has shown to provide the best performance in the eyes of the customer. However, identifying the appropriate performance measures as well as selecting the best target value is not an easy task. Target values are sometimes the designer's best guess. In actuality, the quadratic form was chosen by Taguchi because it was both simple, and as it turned out, useful. Further, after the Taylor expansion, higher powers in the series change the loss at the target value by a very small margin, and for practical purposes can be ignored within experimental error.

Quality loss functions can be used for the nominal-the-best, smaller-the-better, and larger-the-better characteristics (Taguchi, G. 1990). The nominal-the-best characteristic applies to a finite target point. There are typically upper and lower specification limits on both sides of the performance target. For example, the plating thickness of a component, the length of a part, and the output current of a resistor at a given input voltage are nominal-the-best characteristics.

A smaller-the-better output response applies to the desire to minimize the result, with the ideal target being zero. For example, the wear on a component, the amount of engine audible noise, the amount of air pollution and the amount of heat loss are smaller-the-better output responses.

The larger-the-better output response maximizes the result, the ideal target being infinity. For example, the strength of materiel and the fuel efficiency are larger-the-better output responses. The loss function offers a way to quantify the benefits achieved by reducing variability around the target. It can help to justify a decision to invest how much to improve a process that is already capable of meeting specifications. As originally proposed by Taguchi, the objective of minimizing the loss to the customer was to improve quality by minimizing the effects of variations in performance while striving to achieve the performance target value. But accomplishing this goal did not necessarily need come not at the expense of eliminating the causes of that variation. The intent was to design robustness to variation that imparted value to the customer without an associated loss (Yao, L et al 1999).

Types of Quality Loss Functions. The quality loss function developed by Taguchi considers three cases including nominal-the-best, smaller-the-better, and larger-the-better. The methodology used to deal with the larger-the-better case is slightly different from that for the smaller-the-better and nominal-the-better cases. In reality, for each quality characteristic there exists some function that uniquely defines the relationship between economic loss and the deviation of the quality characteristic from its target value. The time and resources required to obtain such a relationship for each quality characteristic would represent a considerable investment. Taguchi has found the quadratic representation of the quality loss function to be an efficient and effective way to assess the loss due to deviation of a quality characteristic from its target value. For a product with a target value m, from many customers' point of view, $m \pm \Delta_0$ represents the deviation at which functional failure of the product or component occurs. When a product is manufactured with its quality characteristic at the extremes, at $m + \Delta_0$ or $m - \Delta_0$, some countermeasure (or loss per unit that encompasses must be undertaken by the average customer to mitigate loss. The loss function L (average loss) with a characteristic of nominal-the-best (NTB) is described as the following equation (1).

$$L = k(y - m)^2 \quad k = \frac{A_0}{\Delta_0^2} \qquad (1)$$

Where, k is a proportionality constant and could be the cost of each unit (returned, modified, reworked) divided by the range limits of process variability divided by 2, y is the measure of performance (e.g., output) for a given function, m is the target value of y, and A_0 is the cost of the countermeasure. The loss function can also be determined for cases when the output response is a smaller-the-better response. The formula is a little different, but the procedure is much the same as for the case of nominal-the-best. For the case of smaller-the-better (STB), where the target is zero, the loss function is described as the following equation (2).

$$L = ky^2 \quad k = \frac{A_0}{y_0^2} \qquad (2)$$

Where, A_0 is the consumer loss and y_0 is the consumer tolerance.

For a larger-the-better (LTB) output response where the target is infinity, the loss function can be written as the following equation (3).

$$L = k\frac{1}{y^2} \quad k = A_0 y_0^2 \qquad (3)$$

Assumptions. The following seven assumptions are made to develop a general quality loss function.

A1: The total quality loss ($L_n(x)$) consists of the stakeholders' loss, (i.e., developers', manufacturers' loss, and unknown loss.

A2: If the level of quality equals the target value of the quality (i.e., m), the total quality loss is to be minimum.

A3: If the acquisition phase is production and deployment, the value of shape parameter n is to 2.

A4: The minimum value of a shape parameter is close to zero and the value of the shape parameter of the concept refinement phase of the acquisition phases varies from zero to 1.

A5: When the acquisition phases are the technology development or system development and demonstration phase, the range value of shape parameter varies from greater than one to less than two.

A6: After the production and deployment phase, the value of the shape parameter is greater than two.

A7: The probability distribution of the quality response remains the same regardless of the acquisition phases.

Notation.

C_b: Baseline cost with a constant value

C_s: If the type of quality characteristic is smaller-the-better, this means a proportionality constant of stakeholder's loss per response of quality. Additionally, if the type of quality characteristic is larger-the-better, it means a proportionality constant of developer's or manufacturer's loss per response of quality.

C_l: If the type of quality characteristic is larger-the-better, this means a proportionality constant of developer's or manufacturer's loss per response of quality. Additionally, if the type of quality characteristic is smaller-the-better, it means proportionality constant of stakeholder's loss per response of quality.

n: Shape parameter for representing an acquisition phase of a weapon system ($n > 0$)

x: Response of quality

$L_n(x)$: Total quality loss per piece in case of shape parameter n and quality response x

L_n : Expected quality loss per piece in case of shape parameter n and quality response x

According to the assumption A1 and equation (1), (2) and (3), a general quality loss function can be described as the following equation (4). Equation (4) covers all quality characteristics such as nominal-the-best, smaller-the-better, and larger-the-better.

$$L_n(x) = C_b + C_s x^n + C_l x^{-n} \qquad (4)$$

After applying the assumption A2 into the above equation (4), we can get equation (5) and (6) as the follows. If the response of quality equals to the target value (i.e., m), the total quality loss is to be zero (equation (5)) and the result of differentiation for the response of quality having the target value (i.e., m) is also to be zero as the equation (6).

$$L_n(m) = C_b + C_s m^n + C_l m^{-n} = 0 \qquad (5)$$

$$L_n'(m) = nC_s x^{n-1} - nC_l x^{-n-1} = 0 \qquad (6)$$

If we input the specific value of n to the equation (5) and (6), we get the general loss function as the follows. If the value of n equals to 1, we get the following results.

$$L_1(m) = C_b + C_s m^1 + C_l m^{-1} = 0 \qquad (7)$$

$$L_1'(m) = C_s m^0 - C_l m^{-2} = 0 \qquad (8)$$

After solving the above equation (7) and (8), we get the following results.

$C_l = C_s m^2$, $C_b = -2C_s m$.

If n equals to 2, we get the following results.

$$L_2(m) = C_b + C_s m^2 + C_l m^{-2} = 0 \qquad (9)$$

$$L_2'(m) = 2C_s m - 2C_l m^{-3} = 0 \qquad (10)$$

After solving the equation (9) and (10), we get the following results.

$C_l = C_s m^4$, $C_b = -2C_s m^2$.

After iterating in the above manner, we can generate a quality loss function as shown in Table 1.

Table 1. Results of iterative process for generating a quality loss function

n	C_l	C_b	$L_n(x)$
1	$C_l = C_s m^2$	$C_b = -2C_s m^1$	$L_1(x) = -2C_s m^1 + C_s x^1 + C_s m^{2\times 1} x^{-1}$
2	$C_l = C_s m^4$	$C_b = -2C_s m^2$	$L_2(x) = -2C_s m^2 + C_s x^2 + C_s m^{2\times 2} x^{-2}$
3	$C_l = C_s m^6$	$C_b = -2C_s m^3$	$L_3(x) = -2C_s m^3 + C_s x^3 + C_s m^{2\times 3} x^{-3}$
4	$C_l = C_s m^8$	$C_b = -2C_s m^4$	$L_4(x) = -2C_s m^4 + C_s x^4 + C_s m^{2\times 4} x^{-4}$
n	$C_l = C_s m^{2n}$	$C_b = -2C_s m^n$	$L_n(x) = -2C_s m^{2n} + C_s x^n + C_s m^{2n} x^{-n}$

As shown in the last row of the above Table 1, we present the general quality loss function, detailed as follows, equation (11).

$$\begin{aligned} L_n(x) &= -2C_s m^n + C_s x^n + C_s m^{2n} x^{(-n)} \\ &= -2C_s m^n + C_s x^n (1 + m^{2n} x^{(-2n)}) \end{aligned} \qquad (11)$$

Shapes of Quality Loss Function. As the value of n (shown in equation (11) for the general quality loss function) changes, the shapes of quality loss function also change. To illustrate these changes, we plot the value of quality loss versus the response of quality with C_s =2 and m =3, Figure 5, according to the change of the value of n.

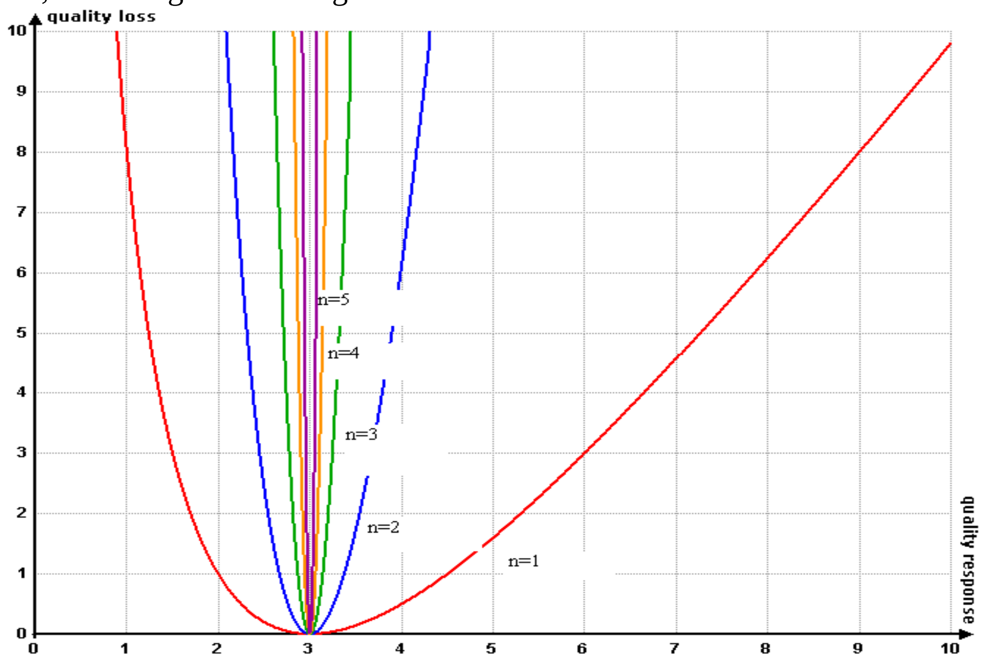

Figure 4. Shapes of the general quality loss function

As shown in Figure 4, the quality loss function with red line is for n = 1, the blue line is for n = 2, the green line is for n = 3, and so on. By plotting the loss functions with different values of n, we observe the width of the quality loss function depends on the value of n. In order words, the larger the value of n, the narrower the width of the quality loss function.

In order to clearly see the proportionality between the quality losses as the value of n changes, we calculate all the related values in Table 2.

Table 2. Values of quality loss according to the value of n

Response of quality (x)	n =1	n =2	n =3	n =4	n =5
1.00	8.00	128.00	1,352.00	12,800.00	117,128.00
1.50	3.00	40.50	330.75	2,278.13	14,595.20
2.00	1.00	12.50	90.25	528.13	2,782.56
2.50	0.20	2.42	16.56	90.05	432.64
3.00	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
3.50	0.14	1.72	11.76	63.57	303.29
4.00	0.50	6.13	42.78	239.26	1,191.33
4.50	1.00	12.50	90.25	528.13	2,782.56
5.00	1.60	20.48	153.66	947.00	5,315.79
5.50	2.27	29.86	233.51	1,520.47	9,117.15
6.00	3.00	40.50	330.75	2,278.13	14,595.20

Quality loss function	$L_n(x) = -2\times 2\times 3^n + 2\times x^n + 2\times 3^{2n} \times x^{-n}$				
Baseline cost $-2\times C_s \times m^n$ $= -2\times 2\times 3^n$	-12	-36	-108	-324	-972

Expected Quality Loss

Now suppose that the probability density function of X is normal with mean μ and variance σ^2. The probability density function of X will be of the following form, equation (23).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2}),\ -\infty \le x \le \infty \qquad (12)$$

The expected loss per item is calculated according to equation (24).

$$E[L_x(n)] \equiv L_n = \int_{-\infty}^{\infty} L_x(n) f(x) dx, \qquad (13)$$

Where $f(x)$ is the probability density function of the normal random variable.

By substituting general quality loss function and probability density function into equation (24), equation (24) can be rewritten as the following equation (25).

$$\begin{aligned} L_n &= \int_{-\infty}^{\infty} L_x(n) f(x)dx = \int_{-\infty}^{\infty} C_s(-2m^n + x^n + m^{2n}x^{(-n)})\frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx \\ &= \int_{-\infty}^{\infty} C_s(-2m^n)\frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx + \int_{-\infty}^{\infty} C_s x^n \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx \\ &+ \int_{-\infty}^{\infty} C_s m^{2n} x^{(-n)} \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx \\ &= L_{n1} + L_{n2} + L_{n3}, \end{aligned} \qquad (14)$$

Where

$$L_{n1} = \int_{-\infty}^{\infty} C_s(-2m^n)\frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx = -2C_s m^n \qquad (15)$$

$$L_{n2} = \int_{-\infty}^{\infty} C_s x^n \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx \qquad (16)$$

$$L_{n3} = \int_{-\infty}^{\infty} C_s m^{2n} x^{(-n)} \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx \qquad (17).$$

Because it is impossible for the above equation (27) and (28) to be integrated as a closed form, we adopt Taylor series expansion as the following. Taylor series for x^n and x^{-n} at target value of x (i.e., m) is the following.

$$x^n = \sum_{k=0}^{\infty} \frac{f^{(k)}(m)}{\Pi k}(x-m)^k = \sum_{k=0}^{n} \frac{f^{(k)}(m)}{\Pi k}(x-m)^k + R_n$$

$$= \sum_{k=0}^{n} \frac{1}{\Pi k} \frac{\Pi(n)}{\Pi(n-k)}(m)^{n-k}(x-m)^k + R_n$$

$$x^{-n} = \sum_{k=0}^{\infty} \frac{f^{(k)}(m)}{\Pi k}(x-m)^k = \sum_{k=0}^{n} \frac{f^{(k)}(m)}{\Pi k}(x-m)^k + R_n$$

$$= \sum_{k=0}^{n} \frac{1}{\Pi k}(-1)^k \frac{\Pi(n-1+k)}{\Pi(n-1)}(m)^{-n-k}(x-m)^k + R_n$$

R_n : Error after n terms

By ignoring terms higher than the fourth order, we get the following form (29) and (30).

$$x^n \approx \sum_{k=0}^{4} \frac{1}{\Pi k} \frac{\Pi(n)}{\Pi(n-k)}(m)^{n-k}(x-m)^k, \qquad (18)$$

$$x^{-n} \approx \sum_{k=0}^{4} \frac{1}{\Pi k}(-1)^k \frac{\Pi(n-1+k)}{\Pi(n-1)}(m)^{-n-k}(x-m)^k \qquad (19)$$

After substituting equation (29) and (30) into equation (25), we get the following results.

$$L_{n1} = -2C_s m^n$$

$$L_{n2} = \int_{-\infty}^{\infty} C_s x^n \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})dx$$

$$\approx C_s \int_{-\infty}^{\infty} \sum_{k=0}^{4} \frac{1}{\Pi k} \frac{\Pi(n)}{\Pi(n-k)}(m)^{n-k}(x-m)^k \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})dx$$

$$= C_s\{m^n + nm^{n-1}(E(X)-m) + \frac{n(n-1)}{2} m^{n-2}(E(X^2) - 2mE(X) + m^2)$$

$$+ \frac{n(n-1)(n-2)}{6} m^{n-3}(E(X^3) - 3mE(X^2) + 3m^2E(X) - m^3)$$

$$+ \frac{n(n-1)(n-2)(n-3)}{24} m^{n-4}(E(X^4) - 4mE(X^3) + 6m^2E(X^2) - 4m^3E(X) + m^4)\}$$

$$L_{n3} = \int_{-\infty}^{\infty} C_s m^{2n} x^{(-n)} \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})dx$$

$$\approx C_s m^{2n} \int_{-\infty}^{\infty} \sum_{k=0}^{4} \frac{1}{\Pi k} (-1)^k \frac{\Pi(n-1+k)}{\Pi(n-1)} (m)^{-n-k} (x-m)^k \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})dx$$

$$= C_s m^n \{1 - nm^{-1}(E(X)-m) + \frac{n(n+1)}{2} m^{-2}(E(X^2) - 2mE(X) + m^2)$$

$$- \frac{n(n+1)(n+2)}{6} m^{-3}(E(X^3) - 3mE(X^2) + 3m^2E(X) - m^3)$$

$$+ \frac{n(n+1)(n+2)(n+3)}{24} m^{-4}(E(X^4) - 4mE(X^3) + 6m^2E(X^2) - 4m^3E(X) + m^4)\}$$

Therefore, the expected quality loss in case of the normal distribution of quality characteristic is of the following form, equation (31).

$$L_n = C_s\{n^2 m^{n-2}(E(X^2) - 2mE(X) + m^2)$$

$$- n^2 m^{n-3}(E(X^3) - 3mE(X^2) + 3m^2E(X) - m^3) \qquad (20)$$

$$+ \frac{(n^4 + 11n^2)}{12} m^{n-4}(E(X^4) - 4mE(X^3) + 6m^2E(X^2) - 4m^3E(X) + m^4)\}$$

Where,

$$E(X^n) = \int_{-\infty}^{\infty} x^n \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2} dx$$

$$E(X^0) = 1$$

$$E(X^1) = \mu$$

$$E(X^2) = \mu^2 + \sigma^2$$

$$E(X^3) = \mu^3 + 3\mu\sigma^2$$

$$E(X^4) = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$$

In order to show the trend of the expected quality loss according to the position of the target value, we consider three cases of the mean of quality output, by using a numerical example.

Case 1: the target value of the quality characteristic is equal to the mean of quality. Case 2: the target value of the quality characteristic is greater than the mean of quality. Case 3: the target value is less than the mean of quality.

Before suggesting the results of the application, we should assume the inputs for demonstrating the trend of the expected quality loss as indicated in Table 3.

Table 3. Data for application with normal distribution

Given data for the general quality loss function	
• Baseline cost(C_b) :	$-2C_s m^n$

• Cost incurred in case of smaller-the-better(C_s) :	0.3
• Cost incurred in case of larger-the-better(C_l) :	$C_s m^{2n}$
Given data for the normal distribution	
• Mean of quality(μ) :	10
• Variance of quality(σ^2)	0.25
• *n*th moment of the probability distribution is given by the `Riemann-Stieltjes integral`	1^{st} :10 2^{nd} : 100.25 3^{rd} : 1,007.5 4^{th} : 10,150.1875
Three cases	
• Case 1 : $m = \mu$:	10
• Case 2 : $m > \mu$:	11
• Case 3 : $m < \mu$:	9

First, consider case 1 and observe the trend of expected quality loss as the value of *n* varies, through a numerical example. After substituting the data from Table 5 into equation (31), we obtain the expected quality loss, per equation (32).

$$\begin{aligned} L_n &= C_s(m^{n-2}n^2\sigma^2 + (n^4 + 11n^2)\sigma^4 / 4) \\ &= 0.3(0.25\times 10^{n-2} n^2 + 0.25^2 \times (n^4 + 11n^2)/4) \end{aligned} \quad (21)$$

For case 2 and case 3, after applying the same method used in equation 32, we obtain the expected values respectively. In order to compare the expected quality loss among three cases, we need to display the expected quality loss to the value of *n* as shown in Figure 5.

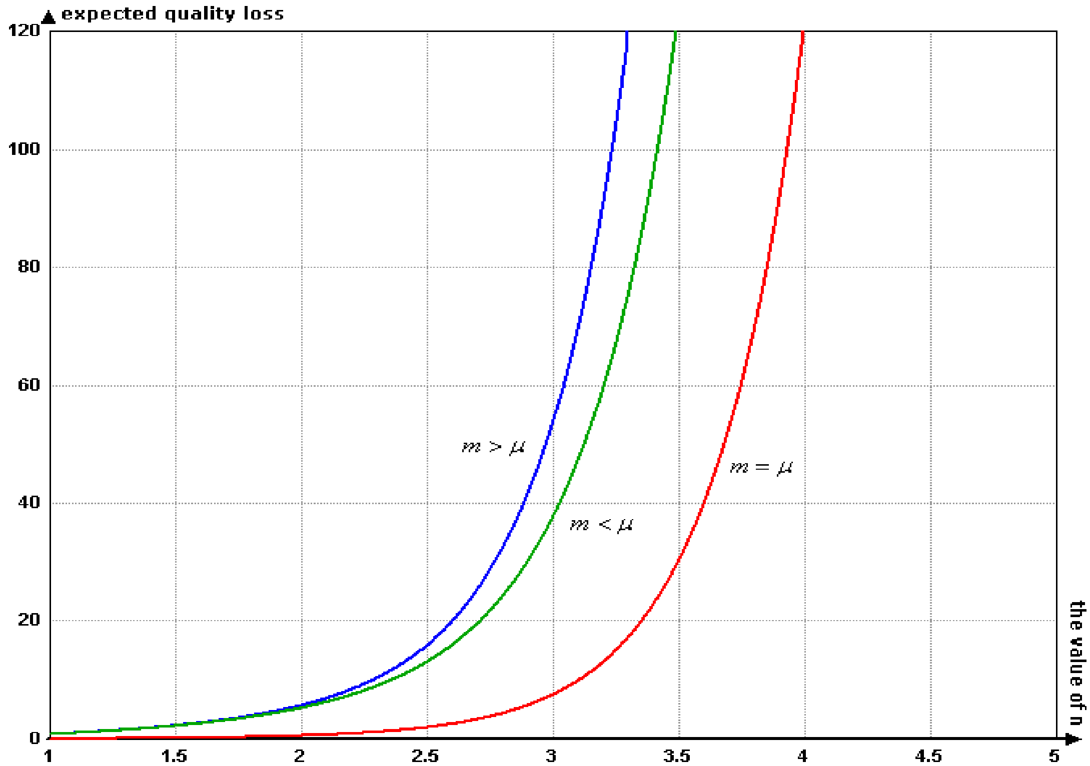

Figure 5. Expected quality loss with normal distribution

The expected quality loss function for case 1 is shown by the red line is, case 2 by the blue line, and case 3 by the green line. By plotting the expected quality loss functions having the different values *n*, we show that the amount of the expected quality loss depends upon the value of *n*, regardless of the position of the target value. In order words, if the value of *n* is increasing, then the slope of the function is increasing.

The amount of the expected quality loss change is proportional to the value of *n*. We show all the related values in Table 4.

Table 4. Expected value of quality loss with normal distribution

n	case 1 ($m = \mu$)	case 2 ($m > \mu$)	case 3 ($m < \mu$)
1	0.06375	0.84468	0.841435
2	0.58125	5.72216	5.29792
3	7.59375	53.9438	37.7437
4	122.025	847.425	439.425
5	1879.22	14126.7	5831.72

Conclusion

A general quality loss function having a shape parameter is developed, which is applicable to evaluate the expected quality loss for quality characteristics such as nominal-the-best, smaller-the-better, and larger-the-better. Additionally, we present an appropriate range of shape parameter values in the proposed general quality loss function to accommodate the impacts of upgrades to fielded systems.

By plotting the loss functions with different values of shape parameter n, we show that the width of the quality loss function depends upon the value of n. In order words, if the value of n is increasing, then the slope of the expected quality loss function is increasing. When we calculate the expected quality loss, we consider the normal probability distribution. Similar results are obtained with the exponential distribution, truncated exponential distribution, and truncated normal distribution. In order to show the applicability of the proposed general quality loss function to the periodicity of upgrading fielded systems, we present the quality loss function and demonstrate a process for determining the acceptance level of periodicity through numerical examples.

Therefore, the proposed general quality loss function can be used to justify a decision to release a product, and determining a specification limit on the release dates that minimizes the expected quality loss.

The limitations of this study are the following: (1) we adopt Taylor series expansion for the general quality loss function. Due to this, the expected quality loss using the proposed function has nominal errors. (2) Since we have difficulty in obtaining actual data for quality loss associated with upgrading products and periodicity, we cannot present a validation of shape parameter value, *n*.

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Biography

Gary Langford is a lecturer in the Systems Engineering Department at the Naval Postgraduate School in Monterey, California. His research interests include the theory of systems engineering and its application to commercial and military competitiveness. Mr. Langford founded and ran five corporations – one NASDAQ listed. He was a NASA Ames Fellow. He received an A.B. in astronomy from UC Berkeley, and an M.S. in physics from Cal State Hayward. He is an INCOSE member.