## Computer Engineering 111

Test 2
October 26, 2009

## Name

$\qquad$

Nine problems, 100 points.
Closed books, closed notes, no calculators. You would be wise to read all problems before beginning, note point values and difficulty of problems, and budget your time accordingly. The order you should do the problems might be different than the order in which they appear.

Please do not open the test until I tell you to.

"This test sounds like fun!"

Good luck! Have fun!
-

1. ( 16 points) Consider the circuit below:


Write F the way the circuit is drawn:

$$
\mathrm{F}=
$$

Now rewrite F in minimal SOP form. You can do this by using the truth table and Kmap provided, by simplifying and rearranging using the rules of Boolean Algebra, or by some combination of the two approaches. Show your work.

$$
\text { (minimal SOP) } \mathrm{F}=
$$

```
ABC F
```

000

001
010
011
100
101
110
111

| $A B C$ | 00 | 01 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |
| 1 |  |  |  |  |

2. (12 points) Find the minimal and canonical SOP implementations of F as given in the truth table below. Use your choice of methods, but show your work. Then do the canonical POS form and the minterm and Maxterm form below.

ABC F
0001
0010
0100
0111
1001
1011
1100
1111

| $A B C$ | 00 | 01 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |
| 1 |  |  |  |  |

The minimal SOP form of F is $\mathrm{F}=$ $\qquad$ .

The canonical SOP form of F is $\mathrm{F}=$ $\qquad$ .

The canonical POS form of F is $\mathrm{F}=$ $\qquad$ .

Write F in terms of minterms, and again in Maxterms, i.e.:
$\mathrm{F}=\Sigma \mathrm{m}($
$\mathrm{F}=\Pi \mathrm{M}($
3. (11 total points) (Careful, this one is tricky with the XS 3 and the don't cares.)

Your input (ABCD) is an XS3 signal and your output is:

$$
\mathrm{F} \quad=\quad 1 \quad \text { if the input is a digit in today's date: } 0,1,2,6 \text { or } 9 .
$$

$0 \quad$ Otherwise.
Write F in terms of minterms, and again in Maxterms, i.e.:
$\mathrm{F}=\Sigma \mathrm{m}($
$\mathrm{F}=\Pi \mathrm{M}(\quad)$ $\left.\mathrm{Mx}^{( }\right)$
Write the expression for F in minimal NAND-NAND logic, solving by 4-variable K-map. Don't forget to properly indicate the don't-care's and use them, above and below!
Truth Table for Problem 3
A B CD F
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
NAND-NAND $\mathrm{F}=$

| AB | 00 | 01 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| 00 |  |  |  |  |
| 01 |  |  |  |  |
| 11 |  |  |  |  |
| 10 |  |  |  |  |

4. (4 points) Write down the number "7" in the following codes:
"Natural" BCD:

XS3:
5. (14 points)

Consider the device below:

a) (4 points) If $\mathrm{A}=1010$ and $\mathrm{B}=1010$ and $\mathrm{cin}=1$, give all outputs.
b) ( 4 points) If $A=1110$ and $B=1101$ and $\operatorname{cin}=0$, give all outputs.
c) ( 4 points) If $A=1011$ and $B=0011$ and $\operatorname{cin}=0$, give all outputs.
d) (2 point) If our inputs are assumed to be 4-bit 2's complement, are any of these answers wrong? If so, which and why?
6. ( 15 points) Your input is an BCD signal and your output is:

$$
\begin{array}{ll} 
& 1 \\
\text { F } \quad \text { if the input is divisible by } 3 \\
0 & \text { otherwise. }
\end{array}
$$

Find the minimal SOP form. Use your don't cares to minimize logic (don't just put 0's there).

$$
F=
$$

| Decimal | ABCD F |
| :---: | :---: |
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |
| 10 | 1010 |
| 11 | 1011 |
| 12 | 1100 |
| 13 | 1101 |
| 14 | 1110 |
| 15 | 1111 |


| AB | 00 | 01 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| 00 |  |  |  |  |
| 01 |  |  |  |  |
| 11 |  |  |  |  |
| 10 |  |  |  |  |

7. (6 points)

Explain what device this table is describing:
Inputs

| I3 3 | I2 | I1 | I0 | Y1 | Y0 | V |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | X | X | X | 1 | 1 | 1 |
| 0 | 1 | X | X | 1 | 0 | 1 |
| 0 | 0 | 1 | X | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(4 points)
This device is a
(1 point)
The reason the X's appear in the input portion is:
(1 point)
The V output signifies
8. (12 points: 6 for (a) and 6 for (b).) Solve for the minimal SOP expression of $F=\bar{W} \bar{X} Y Z+\bar{W} X Y+\bar{W} Y+X \bar{Y}+X Y$
a) By using Boolean Algebra manipulations
b) By using this K-map.

A blank truth table is on the following page, for your use in preparing to fill in this Kmap.

| WX | YZ | 00 | 01 | 11 |
| :--- | :--- | :--- | :--- | :--- |
| 00 |  |  |  | 10 |
| 01 |  |  |  |  |
| 11 |  |  |  |  |
| 10 |  |  |  |  |

Blank truth table for problem 8.
W X YZ F
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
9. (12 points)

Re-implement the diagram shown (by sketching the correct logic circuits ) with exactly:

a) (4 points)

One NAND gate.
b) (4 points)

One 3-8 Decoder
One OR gate (with all the inputs you want)

c) (4 points)

One 8-1 Multiplexer


Sketch your solutions on both parts! At your option, you can use the truth table (on the next page), make a K-map (on the next page), use Boolean Algebra, or your knowledge of the standard use of NAND-NAND logic to compose your answer. (The latter is easiest.) For this problem, it is acceptable to sketch the correct solutions without using the truth table, K-map, or otherwise showing any work.

Optional worksheet for problem 6. Please write your final answer clearly in the space provided on the previous page. The use of a K-map (or any of this stuff) is optional, and I think the problem is easier without it, if you know the relationship between NANDNAND and the requested forms of logic.

| ABC | 00 | 01 | 11 | 10 |
| :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |
| 1 |  |  |  |  |

Here are the postulates and theorems of Boolean Algebra. (Taken from http://www.ee.scu.edu/classes/1999winter/elen021/supp/BooleanAlgebra.html .)You are NOT required to cite them in your derivations, but they are here in case you find them helpful. You may well find you can solve the problem without needing to look back and forth at these.

Assume A, B, and $\mathbf{C}$ are logical states that can have the values $\mathbf{0}$ (false) and $\mathbf{1}$ (true). " + " means OR, " $\cdot$ " means AND, and ${ }^{\text {NOT }}[\mathrm{A}]$ means NOT A.

## Postulates

| $(1)$ | $\mathbf{A}+\mathbf{0}=\mathbf{A}$ | $\mathbf{A} \cdot \mathbf{1}=\mathbf{A}$ | identity |
| :--- | :--- | :--- | :--- |
| $(2)$ | $\mathbf{A}+{ }^{\mathrm{NOT}}[\mathbf{A}]=\mathbf{1}$ | $\mathbf{A} \cdot{ }^{\mathrm{NOT}}[\mathbf{A}]=\mathbf{0}$ | complement |
| $(3)$ | $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$ | $\mathbf{A} \cdot \mathbf{B}=\mathbf{B} \cdot \mathbf{A}$ | commutative law |
| $(4)$ | $\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C}$ | $\mathbf{A} \cdot(\mathbf{B} \cdot \mathbf{C})=(\mathbf{A} \cdot \mathbf{B}) \cdot \mathbf{C}$ | associative law |
| $(5)$ | $\mathbf{A}+(\mathbf{B} \cdot \mathbf{C})=(\mathbf{A}+\mathbf{B}) \cdot(\mathbf{A}+\mathbf{C})$ | $\mathbf{A} \cdot(\mathbf{B}+\mathbf{C})=(\mathbf{A} \cdot \mathbf{B})+(\mathbf{A} \cdot \mathbf{C})$ | distributive law |

## Theorems

| (6) | $\mathbf{A}+\mathbf{A}=\mathbf{A}$ | $\mathbf{A} \cdot \mathbf{A}=\mathbf{A}$ |  |
| :---: | :---: | :---: | :---: |
| (7) | A $+1=1$ | A $\cdot 0=0$ |  |
| (8) | $\mathbf{A}+(\mathbf{A} \cdot \mathbf{B})=\mathbf{A}$ | $\mathbf{A} \cdot(\mathbf{A}+\mathbf{B})=\mathbf{A}$ |  |
| (9) | $\mathbf{A}+\left({ }^{\text {NOT }}[\mathbf{A}] \cdot \mathrm{B}\right)=\mathbf{A}+\mathbf{B}$ | $\mathbf{A} \cdot\left({ }^{\mathrm{NOT}}[\mathbf{A}]+\mathbf{B}\right)=\mathbf{A}$ |  |
| (10) | $\begin{aligned} & (\mathbf{A} \cdot \mathbf{B})+\left({ }^{\mathrm{NOT}}[\mathbf{A}] \cdot \mathbf{C}\right)+(\mathbf{B} \cdot \mathbf{C})=(\mathbf{A} \cdot \\ & \mathbf{B})+\left({ }^{\text {NOT }}[\mathbf{A}] \cdot \mathbf{C}\right) \end{aligned}$ | $\begin{aligned} & \mathbf{A \cdot ( \mathbf { B } + \mathbf { C } ) = ( \mathbf { A } \cdot \mathbf { B } )} \\ & +(\mathbf{A} \cdot \mathbf{C}) \end{aligned}$ |  |
| (11) | ${ }^{\mathrm{NOT}}[A+B]={ }^{\mathrm{NOT}}[A] \cdot{ }^{\mathrm{NOT}}[B]$ | ${ }^{\mathrm{NOT}}[\mathbf{A} \cdot \mathbf{B}]={ }^{\mathrm{NOT}}[\mathbf{A}]+$ | de Morgan's theorem |

This page is an extra sheet of scratch paper. Please don't use it for showing solutions, but refer to it on the problem page if you have work here that you are required to show on the problem, or that you otherwise believe needs to be considered in grading. You are permitted to tear off this sheet if there's nothing on here that you think needs to be considered.

