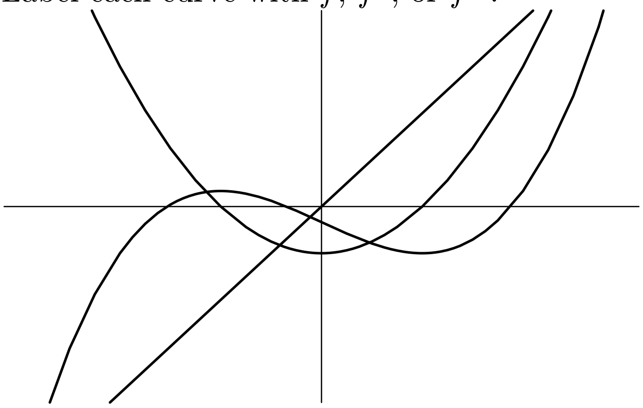
Exam 3

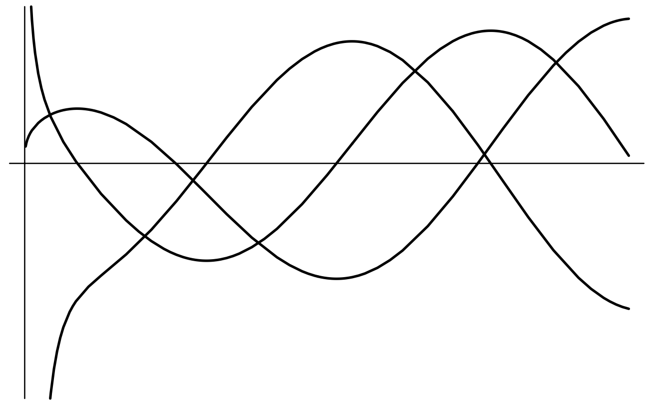
Math 8 DD
October 29, 1993

Name________________________________

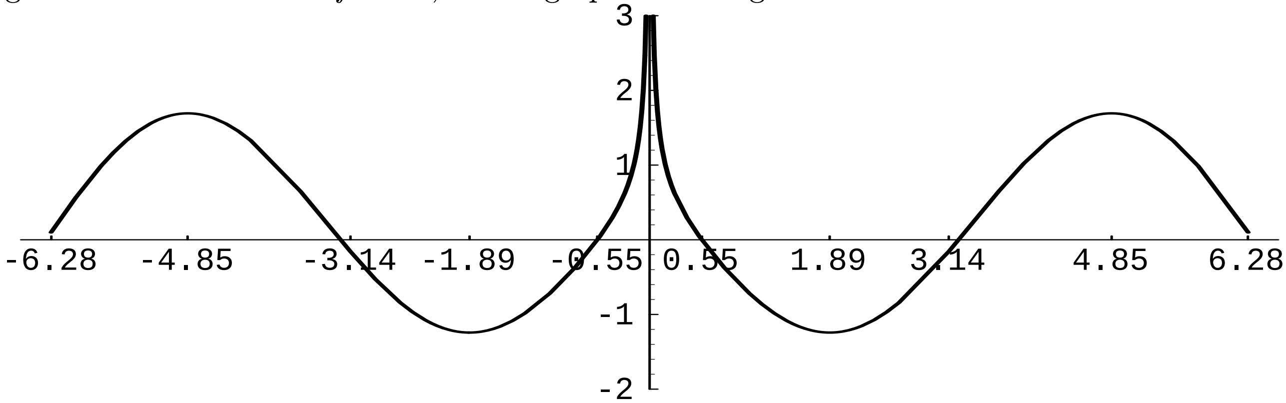
Rec. Section________________

1. (12 pts) Each plot shows the graphs of a function f, its first derivative $f\,'$, and its second derivative $f\,''$. Label each curve with f, $f\,'$, or $f\,''$.





2. WARNING! Read this question carefully! The graph shown is the *derivative* of a function f. You are to give information about f itself, whose graph is NOT given.



(a) (6pts) The critical numbers for f in the interval $[-2\pi, 2\pi]$ are:
(b) (6 pts)Intervals on which f is increasing are:
(c) (6 pts) Intervals on which the graph of f is concave down are:

3. (12 pts) Let $f(x) = x^3 + b\,x^2 + c\,x$. Find a relationship between b and c such that the graph of f has a horizontal tangent line at its inflection point.
4. Sketch the graph of a function f having the following characteristics.
(a) (5 pts) $f(0) = f(2) = 0$, $f\,'(x) < 0$ if $x < 1$, $f\,'(1) = 0$, $f\,'(x) > 0$ if $x > 1$, $f\,''(x) > 0$.
(b) (5 pts) f is continuous for all x, $f(1) = 0$, $f(x)$ is decreasing for $x < 1$, f is not differentiable at $x = 1$, $f(x)$ is decreasing for $x > 1$, the graph of $y = f(x)$ is concave down for $x \neq 1$.
5. (15 pts) The sum of the circumference and the height of a right circular cylinder is 108 inches. Find the dimensions (radius and height) for which the volume of such a cylinder is maximum. The formula for the volume is $V = \pi\,r^2 h$, and the circumference is $2\,\pi\,r$. Ξ

6. The derivative of $f(x) = x^{1/3}\cos x$ is $f'(x) = \dfrac{\cos x - 3x\,\sin x}{3\,x^{2/3}}$. This means that $f'(x) = 0$ whenever $\cos x - 3x\,\sin x = 0$.

(a) (6 pts) Use Newton's Method 3 times, starting wih $x_0 = 0.5$, to approximate one of the zeros of $\cos x - 3\,x\,\sin x$.

$x_1 =$______________ $x_2 =$______________ $x_3 =$______________

(b) (6 pts) Use your calculator to plot the graph of $f(x) = x^{1/3}\cos x$ for $0 \le x \le 5$, sketch the plot, and label the point on the graph which corresponds to the value you computed in part (a).

7. (21 pts) True or False. If you answer False, you must give a reason. Often, a good reason is a counterexample. In any case, express yourself clearly. Watch for subtelties.

T F If the function f is continuous on $[a, b]$, then f has both a maximum and a minimum on $[a, b]$.
T F The function $x^2 - 4\,x + 4$ has a relative minimum equal to 2.
T F $y = x$ is a slant asymptote for the graph of $f(x) = \dfrac{x^2 - 3\,x + 4}{x - 2}$.
T F Rolle's Theorem is a special case of the Mean Value Theorem.
T F If $y = f(x) = x^2 + 2\,x$, then $dy = (2\,x + 2)\,dx = (2\,x + 2)\Delta x$.
T F If $y = f(x) = x^2 + 2\,x$, then $\Delta y = (2\,x + 2)\,dx = (2\,x + 2)\Delta x$.
T F If $y = f(x) = x^2 + 2\,x$, then $\Delta y - dy = \epsilon\,\Delta x$, where ϵ is a constant.