Note: This time, the True/False questions are scattered throughout the exam. In some cases the T/F question is related to the problem that follows, so be alert. Each T/F is worth 3 points, and the usual rules apply. For full credit, responses of "False" must be accompanied by a reason.

1. T F If \( f'(x) = g'(x) \), then \( f(x) = g(x) \).

2. (18 pts) A function \( f \) has second derivative \( f''(x) = 6x + 2 \). The graph of \( y = f(x) \) passes through the point \((1, 3)\), and the equation of the tangent line to the graph of \( y = f(x) \) at \((1, 3)\) is \( y = 4x - 1 \). Find \( f(x) \).

3. T F For \( f(x) \geq 0 \), the geometric interpretation of \( \int_a^b f(x) \, dx \) is the area below the graph of \( y = f(x) \) and above the x-axis between \( x = a \) and \( x = b \).

4. (12 pts) The graph below is made up of two straight lines and an arc of a circle centered at \((0, 0)\). Write a definite integral or a combination of definite integrals which gives the area under the curve for \( 0 \leq x \leq 5 \). Do not evaluate the integral(s).

5. T F A function that is continuous on a closed bounded interval has both a maximum and a minimum on that interval.

6. (18 pts) Consider the area below the graph of \( y = x^3 - 9x^2 + 20x \) and above the x-axis between \( x = 0 \) and \( x = 4 \). Partition the interval \([0, 4]\) into 4 equal subintervals.

(i) \( x_0 = \) \( x_1 = \) \( x_2 = \) \( x_3 = \) \( x_4 = \)

(ii) For each subinterval, choose \( c_i \) so that \( f(c_i) \) is the minimum for that subinterval.

\( c_1 = \) \( c_2 = \) \( c_3 = \) \( c_4 = \)

(iii) For each subinterval, choose \( c_i \) so that \( f(c_i) \) is the maximum for that subinterval.

\( c_1 = \) \( c_2 = \) \( c_3 = \) \( c_4 = \)

(iv) Draw a large graph showing the function and the 4 circumscribed rectangles. (Label things so I know you know what you are doing.)
7. T F $\int_a^b f(x) \, dx = f(b) - f(a)$.

8. (8 pts) Write the definite integral represented by \( \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \sqrt{c_i^2 + 4 \, \Delta x_i} \) on \([0, 3]\), and explain the meaning of "\( \|\Delta\| \to 0"."

9. T F $\int_a^b f(t) \, dt = \int_a^b f(u) \, du$. (That is, the name of the variable in a definite integral does not matter.)

10. T F $\int_a^b f(x) \, dx + \int_b^c f(x) \, dx = \int_a^c f(x) \, dx$ if $a \leq b \leq c$, but not otherwise.

11. (12 pts) Let the function $f$ be defined by $\int_1^x \frac{1}{t} \, dt$. (At this stage of your calculus career, $\frac{1}{t}$ cannot be integrated, so don’t try.) In what follows, $a$ and $b$ are positive numbers.
   (i) Find $f'(x)$.
   (ii) Show that $f(a)$ can be expressed as $\int_b^a \frac{1}{t} \, dt$. Hint: Try the substitution $u = bt$.
   (iii) Use part (ii) and the definition of $f$ to show that $f(ab) = f(a) + f(b)$.

Midpoint Rule:
$$\int_a^b f(x) \, dx \approx \frac{b-a}{n} [f\left(\frac{x_0 + x_1}{2}\right) + f\left(\frac{x_1 + x_2}{2}\right) + \ldots + f\left(\frac{x_{n-1} + x_n}{2}\right)]$$

Trapezoidal Rule:
$$\int_a^b f(x) \, dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)]$$

Simpson’s Rule:
$$\int_a^b f(x) \, dx \approx \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)]$$

12. (14 pts) If you try to approximate $\int_0^\pi \frac{\sin x}{x} \, dx$ using the Trapezoidal Rule or Simpson’s Rule on your calculator, you get an error message and the calculation stops. The Midpoint Rule, however, works fine.
   (i) Why does the Midpoint Rule work for this integral and why do the other two rules fail to work?
   (ii) Estimate the integral using the Midpoint Rule and $n = 20$. 

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